Math 222 Final Exam B December 18, 2019

Instructions: Wait to open the exam until instructed to do so. Then answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page. You will have 1 hour and 50 minutes to complete this exam.

Question	Points	Score
1	15	
2	10	
3	20	
4	15	
5	10	
6	20	
7	20	
8	15	
9	15	
Total:	140	

Name: _____

Recitation Instructor: _____

Recitation Time: _____

1. Let $\mathbf{u} = \langle 1, 0, 1 \rangle$ and $\mathbf{v} = \langle 1, 0, -1 \rangle$.

   (a) (5 points) What is the angle between $\mathbf{u}$ and $\mathbf{v}$?

   (b) (5 points) Compute the volume of the parallelepiped spanned by $\mathbf{u}$, $\mathbf{v}$ and $\langle 1, 1, 1 \rangle$.

   (c) (5 points) Give a parametric equation passing through $(1, 2, 3)$ in the direction of $\mathbf{u}$.

2. Consider the curve C given by the parametrization

$$\mathbf{r}(t) = \langle \cos t, \sin t, t^2 \rangle \quad \text{for } 1 \leq t \leq 3$$

(a) (5 points) Find the speed of $\mathbf{r}(t)$ as a function of t.

(b) (5 points) Compute the scalar line integral

$$\int_C \sqrt{x^2 + y^2 + 4z}\, ds.$$

3. Calculate the following quantities if they exist. Otherwise, explain why they do not exist. Justify either response.

   (a) (5 points)

$$\lim_{(x,y)\to(0,0)} \frac{xy+y^2}{x^4+y^4}$$

   (b) (5 points) For

$$f(x,y) = x^2 - \cos(xy^2)$$

   compute

$$f_{xy}(x,y)$$

(c) (5 points) For $f(x, y, z) = xy - yz$ find the unit vector in the direction for which f decreases the most starting at the point $(1, 1, 0)$.

(d) (5 points) Find the equation for the tangent plane to the surface

$$xyz = 12$$

at the point $(2, 3, 2)$.

4. Let $\mathcal{D}$ be the square $[0,2] \times [0,2]$ and

$$f(x,y) = xy - x - y$$

(a) (5 points) Find the critical points of $f(x,y)$ in the interior of $\mathcal{D}$.

(b) (5 points) Describe the local behavior of $f(x,y)$ at the critical points found in part (a).

(c) (5 points) Find the maximum and minimum values of f on $\mathcal{D}$.

5. (10 points) Let $\mathcal{W} = [0,1] \times [0,2] \times [0,3]$. Evaluate the triple integral

$$\iiint_{\mathcal{W}} 1 + ye^{yz} \, \mathrm{d}V$$

6. Evaluate the following integrals.

   (a) (10 points) Let $\mathcal{D}$ be the region $1 \leq x^2 + y^2 \leq 9$ and $x \leq 0$. Evaluate

$$\iint_{\mathcal{D}} x \ \mathrm{d}A.$$

(b) (10 points) Let $\mathcal{D}$ be the region $-1 \leq y \leq 1$ and $0 \leq x \leq 1 - y^2$.

$$\iint_{\mathcal{D}} x \ \mathrm{d}A.$$