Math 222 **Practice Exam 1** **February 14, 2019**

Instructions: Wait to open the exam until instructed to do so. Then answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page. You will have 1 hour to complete this exam.

Question	Points	Score
1	30	
2	30	
3	20	
4	20	
Total:	100	

Name: ______________________________

Recitation Instructor: ______________________________

Recitation Time: ______________________________

1. For the following questions, suppose $\mathbf{u} = \langle 0, -1, 1 \rangle$ and $\mathbf{v} = \langle -1, 0, 1 \rangle$.

   (a) (5 points) Evaluate $\mathbf{u} - 3\mathbf{v}$.

   (b) (5 points) Evaluate $\mathbf{u} \cdot \mathbf{v}$.

   (c) (5 points) Find the angle θ between $\mathbf{u}$ and $\mathbf{v}$.

(d) (5 points) Evaluate $\mathbf{u} \times \mathbf{v}$.

(e) (5 points) Find the volume of the parallelepiped with adjacent edges given by $\mathbf{u}$, $\mathbf{v}$ and the standard unit vector $\mathbf{j}$.

(f) (5 points) Find the projection of $\mathbf{v}$ in the direction of $\mathbf{u}$.

2. Solve the problems regarding the points $P = (1, 1, 3)$, $Q = (1, 1, 1)$ and $R = (1, -1, 1)$.

   (a) (8 points) Find a parametric equation for the line ℓ passing through P and Q.

   (b) (7 points) Find the distance between the line in part (a) and the point R.

(c) (8 points) Find the equation for the plane passing through points P, Q and R.

(d) (7 points) Find the equation for the plane which passes through P and Q, and is perpendicular to the plane found in part (c).

3. (a) (10 points) Sketch and describe the trace of the intersection of the plane $y = 3$ with the surface $z^2 - x^2 - y^2 = 16$.

(b) (10 points) The intersections of the surface $ax^2 + by^2 + cz^2 = 1$ with the plane $x = 1$ and the plane $z = 1$ are circles of radius 1. Find the real numbers a, b, c.

4. (a) (10 points) Does the equation $z^2 = x^2 + y^2$ in Cartesian coordinates describe the same surface as $z = r$ in cylindrical coordinates? If yes, explain why. If no, give equations that do.

(b) (10 points) Find an equation of the form $\phi = f(\rho, \theta)$ in spherical coordinates for the surface $z = r$ given in cylindrical coordinates.

Some formulas

$$\begin{aligned}
\mathbf{u} &= \langle u_1, u_2, u_3 \rangle \\
\mathbf{v} &= \langle v_1, v_2, v_3 \rangle \\
\mathbf{u} \times \mathbf{v} &= \langle u_2 v_3 - v_2 u_3, u_3 v_1 - u_1 v_3, u_1 v_2 - u_2 v_1 \rangle
\end{aligned}$$

$$\operatorname{proj}_{\mathbf{u}} \mathbf{v} = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|^2} \mathbf{u}$$

Coordinate systems

Cylindrical

$$\begin{aligned}
x &= r\cos(\theta) \\
y &= r\sin(\theta) \\
z &= z
\end{aligned}$$

$$\begin{aligned}
r &= \sqrt{x^2 + y^2} \\
\tan(\theta) &= \frac{y}{x} \\
z &= z
\end{aligned}$$

Spherical

$$\begin{aligned}
x &= \rho\cos(\theta)\sin(\phi) \\
y &= \rho\sin(\theta)\sin(\phi) \\
z &= \rho\cos(\phi)
\end{aligned}$$

$$\begin{aligned}
\rho &= \sqrt{x^2 + y^2 + z^2} \\
\tan(\theta) &= \frac{y}{x} \\
\cos(\phi) &= \frac{z}{\rho}
\end{aligned}$$