Math 222 Exam 1 February 14, 2019

Instructions: Wait to open the exam until instructed to do so. Then answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page. You will have 1 hour to complete this exam.

Question	Points	Score
1	30	
2	30	
3	20	
4	20	
Total:	100	

Name: _____

Recitation Instructor: _____

Recitation Time: _____

1. For the following questions, suppose $\mathbf{u} = \langle 2, -1, 2 \rangle$ and $\mathbf{v} = \langle 1, 2, -2 \rangle$.

   (a) (5 points) Evaluate $2\mathbf{u} + \mathbf{v}$.

   (b) (5 points) Evaluate $\mathbf{u} \cdot \mathbf{v}$.

   (c) (5 points) Do the vectors $\mathbf{u}$ and $\mathbf{v}$ make an acute, right or obtuse angle? Justify your response.

(d) (5 points) Evaluate $\mathbf{u} \times \mathbf{v}$.

(e) (5 points) Find the area of the parallelogram spanned by $\mathbf{u}$ and $\mathbf{v}$.

(f) (5 points) Find the projection of $\mathbf{v}$ in the direction of $\mathbf{u}$.

2. Solve the problems regarding the points $P = (2, 2, 0)$, $Q = (3, 0, 1)$ and $R = (2, -1, 1)$.

   (a) (8 points) Find a parametric equation for the line ℓ which is parallel to the line passing through P and Q and contains R.

   (b) (7 points) Find the distance between the line in part (a) and the origin.

(c) (8 points) Find the equation for the plane passing through points P, Q and R.

(d) (7 points) Find the distance between the plane found in part (c) and the point $(0, 0, 1)$.

3. (a) (10 points) Sketch and describe the trace of the intersection of the plane $y = -7$ with the surface

$$x^2 - y + z^2 = 16.$$

(b) (10 points) For which values of t are the $x = t$ traces of the equation $x^2 + 2x + y^2 + z^2 = 1$ empty? Describe the graph of this equation.

4. (a) (10 points) Convert the equation $y = zx$ to cylindrical coordinates and use this to show that if a point $P = (a, b, c)$ is on the graph (in Cartesian coordinates), then so is (ta, tb, c) for all positive t.

(b) (10 points) Find an equation in Cartesian coordinates for the equation

$$2\cos\varphi = \rho$$

and describe its graph.

Some formulas

$$\begin{aligned}\mathbf{u} &= \langle u_1, u_2, u_3 \rangle \\ \mathbf{v} &= \langle v_1, v_2, v_3 \rangle \\ \mathbf{u} \times \mathbf{v} &= \langle u_2 v_3 - v_2 u_3, u_3 v_1 - u_1 v_3, u_1 v_2 - u_2 v_1 \rangle\end{aligned}$$

$$\text{proj}_{\mathbf{u}} \mathbf{v} = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|^2} \mathbf{u}$$

Coordinate systems

Cylindrical

$$\begin{aligned}x &= r\cos(\theta) \\ y &= r\sin(\theta) \\ z &= z\end{aligned}$$

$$\begin{aligned}r &= \sqrt{x^2 + y^2} \\ \tan(\theta) &= \frac{y}{x} \\ z &= z\end{aligned}$$

Spherical

$$\begin{aligned}x &= \rho\cos(\theta)\sin(\varphi) \\ y &= \rho\sin(\theta)\sin(\varphi) \\ z &= \rho\cos(\varphi)\end{aligned}$$

$$\begin{aligned}\rho &= \sqrt{x^2 + y^2 + z^2} \\ \tan(\theta) &= \frac{y}{x} \\ \cos(\varphi) &= \frac{z}{\rho}\end{aligned}$$