Math 222 **Exam 2** **March 21, 2019**

Instructions: Wait to open the exam until instructed to do so. Then answer the questions in the spaces provided on the question sheets. Be sure to show your work in every problem and explain your answer. You will have 1 hour to complete this exam.

Question	Points	Score
1	15	
2	20	
3	15	
4	20	
5	10	
6	20	
Total:	100	

Name: ______________________________

Recitation Instructor: ______________________________

Recitation Time: ______________________________

1. Answer the following questions concerning the vector valued function

$$\mathbf{r}(t) = \left\langle \frac{1}{t}, \cos(\pi t), \sin(\pi t) \right\rangle$$

for $t \neq 0$.

(a) (5 points) Evaluate

$$\lim_{t \to 1} \mathbf{r}(t).$$

(b) (5 points) Evaluate $\mathbf{r}''(t)$.

(c) (5 points) Evaluate

$$\int_1^2 \mathbf{r}(t)\,\mathrm{d}t.$$

2. Consider the vector valued function

$$\mathbf{r}(t) = \left\langle t^{3/2}, \frac{3}{2}t, (t-1)^{3/2} \right\rangle$$

for $1 \leq t \leq 2$,

(a) (8 points) Find the arc-length function $s(t)$ of $\mathbf{r}(t)$.

(b) (7 points) What is the length of the curve parametrized by $\mathbf{r}(t)$?

(c) (5 points) Find the arc-length parametrization $\mathbf{r}(s)$.

3. Find the limit, if it exists. If the limit does not exist, explain why.

   (a) (5 points)

$$\lim_{(x,y)\to(0,0)} \frac{|x||y|}{x^2+y^2}$$

   (b) (5 points)

$$\lim_{(x,y)\to(0,0)} \cos\left(\frac{x^3}{x^2+y^2}\right)$$

   (c) (5 points)

$$\lim_{(x,y,z)\to(0,0,1)} \frac{3x+2y+z}{x^2+2y^2+3z^2}$$

4. Evaluate the partial derivatives, if they exist. If they do not exist, explain why.

   (a) (6 points) $f_y(1,1)$ for $f(x,y) = \sin(\pi xy) + x^2 + y^2$.

   (b) (7 points) $f_{xy}(1,1)$ for $f(x,y) = x\ln(y) + x^2 + y^2$.

   (c) (7 points) $f_y(0,0)$ for $f(x,y) = x + 2y|x| - 3y - 4x|y|$

5. Consider the function $f(x, y) = xy + 2$.

   (a) (5 points) Give the linearization $L(x, y)$ of $f(x, y)$ at $(1, -1)$.

   (b) (5 points) Use your result from part (a) to write the equation for the tangent plane to the graph of $f(x, y)$ at $(1, -1)$.

6. Let

$$f(x, y, z) = xe^y + ye^z + z$$

(a) (6 points) Find the gradient of f.

(b) (7 points) Give the equation for the tangent plane of the level surface

$$f(x, y, z) = 1$$

at the point $(1, 0, 0)$.

(c) (7 points) For what unit vector $\mathbf{u}$ is f is decreasing fastest when moving in the direction of $\mathbf{u}$ starting at $(1, 0, 0)$?

Coordinate systems

Cylindrical

$$x = r\cos(\theta)$$
$$y = r\sin(\theta)$$
$$z = z$$

$$r = \sqrt{x^2 + y^2}$$
$$\tan(\theta) = \frac{y}{x}$$
$$z = z$$

Spherical

$$x = \rho\cos(\theta)\sin(\varphi)$$
$$y = \rho\sin(\theta)\sin(\varphi)$$
$$z = \rho\cos(\varphi)$$

$$\rho = \sqrt{x^2 + y^2 + z^2}$$
$$\tan(\theta) = \frac{y}{x}$$
$$\cos(\varphi) = \frac{z}{\rho}$$

Derivative formulas

Directional derivative : $D_{\mathbf{u}}f(x, y) = \nabla f(x, y) \cdot \mathbf{u}$