Math 222 Practice Exam 3 April 18, 2019

Instructions: Wait to open the exam until instructed to do so. Then answer the questions in the spaces provided on the question sheets. Please explain your responses in full detail. You will have 1 hour to complete this exam.

Question	Points	Score
1	20	
2	15	
3	15	
4	15	
5	15	
6	20	
Total:	100	

Name: ______________________________

Recitation Instructor: ______________________________

Recitation Time: ______________________________

1. Consider the function

$$f(x, y) = xy$$

on the domain $\mathcal{D}$ which is a triangle with vertices $(-1, -1), (2, -1)$ and $(-1, 2)$.

(a) (5 points) Find the critical points of f in the interior of $\mathcal{D}$.

(b) (5 points) Describe the local behavior of f near the critical points.

(c) (10 points) Find the global maximum value and the global minimum value for f on $\mathcal{D}$ if they exist. Explain your response.

2. (15 points) Use Lagrange multipliers to find the critical points of the function

$$f(x, y, z) = y^2 - z + x$$

on the unit sphere

$$x^2 + y^2 + z^2 = 1.$$

Identify the global maximum and minimum values of f on the sphere.

3. (15 points) Calculate the integral

$$\iiint_{\mathcal{B}} y^2 z e^{xyz} + x\cos(z)\ \mathrm{d}V$$

where $\mathcal{B} = [-1, 1] \times [0, 1] \times [-1, 0]$.

4. (15 points) Let $\mathcal{D}$ be the region in the first quadrant bounded by the coordinate axes and $y = 1 - x^2$. Evaluate

$$\iint_{\mathcal{D}} 2\pi^2 x \cos(\pi y)\, \mathrm{d}A$$

5. Consider the region $\mathcal{E}$ of points (x, y, z) satisfying

$$x^2 + y^2 \leq z^2, \ x^2 + y^2 + z^2 \leq 4, \ z \geq 0$$

(a) (10 points) Express the triple integral,

$$\iiint_{\mathcal{E}} z \ dV$$

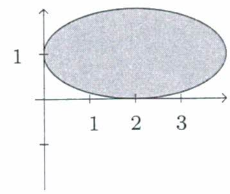
as an iterated integral using spherical coordinates.

(b) (5 points) Evaluate the integral.

6. Let $\mathcal{R}$ be the region inside the ellipse

$$\frac{(x-2)^2}{4} + (y-1)^2 \leq 1$$

which is illustrated below:



(a) (5 points) Assuming $r \geq 0$ and $0 \leq \theta \leq 2\pi$, describe the domain $\mathcal{S}$ in (r, θ)-coordinates which maps onto $\mathcal{R}$ by the transformation

$$T(r, \theta) = (2r\cos(\theta) + 2, r\sin(\theta) + 1).$$

(b) (5 points) Compute the Jacobian of T.

(c) (10 points) Use the change of variables formula to compute the double integral

$$\iint_{\mathcal{R}} x^2 + 4y^2 \, \mathrm{d}A$$

Derivative formulas

$$\text{Directional derivative}: D_{\mathbf{u}} f(x,y) = \nabla f(x,y) \cdot \mathbf{u},$$
$$\text{Discriminant}: D = f_{xx}(x_0,y_0) f_{yy}(x_0,y_0) - f_{xy}(x_0,y_0)^2$$

Coordinate systems

Polar	Cylindrical	Spherical
$x = r\cos(\theta)$	$x = r\cos(\theta)$	$x = \rho\cos(\theta)\sin(\phi)$
$y = r\sin(\theta)$	$y = r\sin(\theta)$	$y = \rho\sin(\theta)\sin(\phi)$
	$z = z$	$z = \rho\cos(\phi)$
$r = \sqrt{x^2+y^2}$	$r = \sqrt{x^2+y^2}$	$\rho = \sqrt{x^2+y^2+z^2}$
$\tan(\theta) = \frac{y}{x}$	$\tan(\theta) = \frac{y}{x}$	$\tan(\theta) = \frac{y}{x}$
	$z = z$	$\cot(\phi) = \frac{z}{\sqrt{x^2+y^2}}$
$\mathrm{d}x\,\mathrm{d}y = r\,\mathrm{d}r\,\mathrm{d}\theta$	$\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = r\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}z$	$\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = \rho^2\sin\phi\,\mathrm{d}\rho\,\mathrm{d}\phi\,\mathrm{d}\theta$

Change of variables

$$T : \mathcal{S} \to \mathcal{R}$$

$$T(u,v) = (x(u,v), y(u,v))$$

$$\mathrm{Jac}(T) = \frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial x}{\partial v}\frac{\partial y}{\partial u}$$

$$\iint_{\mathcal{R}} f(x,y)\,\mathrm{d}x\,\mathrm{d}y = \iint_{\mathcal{S}} f(x(u,v), y(u,v))\ |\mathrm{Jac}(T)|\ \mathrm{d}u\,\mathrm{d}v$$