Math 222 **Practice Final Exam** **May 15, 2019**

Instructions: Wait to open the exam until instructed to do so. Then answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page. You will have 1 hour and 50 minutes to complete this exam.

Question	Points	Score
1	15	
2	10	
3	20	
4	15	
5	10	
6	20	
7	20	
8	15	
9	10	
Total:	135	

Name: ______________________________

Recitation Instructor: ______________________________

Recitation Time: ______________________________

1. Let $\mathbf{u} = \langle -1, 2, 0 \rangle$, $\mathbf{v} = \langle 2, 0, 2 \rangle$ and $\mathbf{w} = \langle 0, 3, 1 \rangle$

   (a) (5 points) Compute the area of the parallelogram spanned by $\mathbf{u}$ and $\mathbf{v}$.

   (b) (5 points) Compute $\cos(\theta)$ where θ is the angle between $\mathbf{v}$ and $\mathbf{w}$.

   (c) (5 points) Give a parametric equation for the line perpendicular to $\mathbf{u}$ and $\mathbf{v}$ and passing through the point $(-1, 2, 3)$.

2. Consider the curve $\mathcal{C}$ given by the parametrization

$$\mathbf{r}(t) = \langle 2t, t^2, t \rangle \quad \text{for } 0 \le t \le 2$$

(a) (5 points) Find the unit tangent vector $\mathbf{T}(t)$ of $\mathbf{r}(t)$.

(b) (5 points) Compute the scalar line integral

$$\int_{\mathcal{C}} \sqrt{2y + xz + 5}\, \mathrm{d}s.$$

3. Calculate the following quantities if they exist. Otherwise, explain why they do not exist. Justify either response.

   (a) (5 points)

$$\lim_{(x,y)\to(0,0)} \frac{x^2+2y^2}{x^2+y^2}$$

   (b) (5 points) For

$$f(x,y,z) = e^{\sqrt{x^2+y^2}} + e^{\sqrt{z^2+y^2}} + 2xz$$

   compute

$$f_{xz}(x,y,z)$$

(c) (5 points) For $f(x, y, z) = y\sin(x) - 4e^z$ find a unit vector pointing in the direction where f increases the fastest, starting at $(0, 3, 0)$.

(d) (5 points) Find the equation for the tangent plane to the surface

$$z = x^2 - y^2$$

at the point $(2, -1, 3)$.

4. Let

$$f(x, y) = x^2 + \cos y$$

and

$$\mathcal{D} = [-1, 1] \times \left[-\frac{\pi}{2}, \frac{\pi}{2}\right].$$

(a) (5 points) Find the critical points of $f(x, y)$ in the interior of $\mathcal{D}$.

(b) (5 points) Describe the local behavior of $f(x, y)$ at the critical points found in part (a).

(c) (5 points) Find the global maximum and minimum values of $f(x, y) = x^2 + \cos y$ on $\mathcal{D}$.

5. (10 points) Let $\mathcal{W} = [0, \pi] \times [-1, 1] \times [2, 3]$. Evaluate the triple integral

$$\iiint_{\mathcal{W}} (3y^2 + z) \sin x \; \mathrm{d}V$$

6. Evaluate the following integrals.

   (a) (10 points) Let $\mathcal{D}$ be the region $0 \leq x \leq 1 - y^2$. Evaluate

$$\iint_{\mathcal{D}} 1 + y^2 \ \mathrm{d}A.$$

(b) (10 points) Let $\mathcal{E}$ be the half ball $x^2 + y^2 + z^2 \leq 1$ with $0 \leq z$ and evaluate

$$\iiint_{\mathcal{W}} z^2 \ \mathrm{d}V.$$

7. Let

$$\mathbf{F} = \langle 2x, 2y, -4z \rangle .$$

(a) (5 points) If $\mathbf{F}$ is a conservative vector field, find a potential. Otherwise, explain why it is not conservative.

(b) (5 points) Let $\mathcal{C}$ be an oriented curve from the origin to $(1, 2, 3)$. Compute

$$\int_{\mathcal{C}} \mathbf{F} \cdot \mathrm{d}\mathbf{r}.$$

(c) (5 points) Verify that the vector field $\mathbf{A} = \langle 3zy, -zx, yx \rangle$ is a vector potential for $\mathbf{F}$, i.e. it satisfies $\mathbf{F} = \text{curl}(\mathbf{A})$.

(d) (5 points) Let $\mathcal{S}$ be the ellipsoid $x^2 + 4y^2 + 9z^2 = 1$ oriented outwardly. Compute the surface integral

$$\iint_{\mathcal{S}} \mathbf{F} \cdot \mathrm{d}\mathbf{S}.$$

State any theorems used in the computation.

8. Consider the annulus $\mathcal{D} = \{(x, y) : 1 \leq x^2 + y^2 \leq 4\}$, the unit circle $\mathcal{C}_1$ oriented counter-clockwise, the radius 2 circle $\mathcal{C}_2$ oriented counter-clockwise (both circles centered at the origin) and the vector field

$$\mathbf{F} = \langle -y, x \rangle .$$

(a) (5 points) Compute the line integral

$$\int_{\mathcal{C}_1} \mathbf{F} \cdot d\mathbf{r}$$

(b) (5 points) Using polar coordinates, find the area of $\mathcal{D}$ by computing the double integral

$$\iint_{\mathcal{D}} \mathrm{d}A$$

(c) (5 points) Using only Green's Theorem and the computations in parts (a) and (b), compute the vector line integral

$$\int_{\mathcal{C}_2} \mathbf{F} \cdot \, \mathrm{d}\mathbf{r}$$

9. Consider the vector field

$$\mathbf{F} = \left\langle e^{x^2+y^2}, y, z^2 - 2xze^{x^2+y^2} \right\rangle$$

and let $\mathcal{E}$ be the box $[0,3] \times [0,2] \times [0,1]$ and $\mathcal{S}$ its boundary oriented outwardly from $\mathcal{E}$.

(a) (5 points) Let $\mathcal{S}_1 = \{(x,y,1) : 0 \leq x \leq 3, 0 \leq y \leq 2\}$ be the rectangular part of $\mathcal{S}$ where $z = 1$. Write the vector surface integral

$$\iint_{\mathcal{S}_1} \mathbf{F} \cdot \, \mathrm{d}\mathbf{S}$$

as an iterated integral. Do **not** evaluate the iterated integral.

(b) (5 points) Using the Divergence Theorem, compute the surface integral

$$\iint_{\mathcal{S}} \mathbf{F} \cdot \, \mathrm{d}\mathbf{S}$$

Coordinate systems

Polar	Cylindrical	Spherical
$x = r\cos(\theta)$	$x = r\cos(\theta)$	$x = \rho\cos(\theta)\sin(\phi)$
$y = r\sin(\theta)$	$y = r\sin(\theta)$	$y = \rho\sin(\theta)\sin(\phi)$
	$z = z$	$z = \rho\cos(\phi)$
$r = \sqrt{x^2+y^2}$	$r = \sqrt{x^2+y^2}$	$\rho = \sqrt{x^2+y^2+z^2}$
$\tan(\theta) = \frac{y}{x}$	$\tan(\theta) = \frac{y}{x}$	$\tan(\theta) = \frac{y}{x}$
	$z = z$	$\cot(\phi) = \frac{z}{\sqrt{x^2+y^2}}$
$\mathrm{d}x\,\mathrm{d}y = r\,\mathrm{d}r\,\mathrm{d}\theta$	$\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = r\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}z$	$\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = \rho^2\sin\phi\,\mathrm{d}\rho\,\mathrm{d}\phi\,\mathrm{d}\theta$

Unit vectors

$$\mathbf{T} = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|} \qquad \mathbf{N} = \frac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$$

Useful area and volume formulas

$$\text{Surface area of sphere of radius } R = 4\pi R^2$$
$$\text{Volume of sphere of radius } R = \frac{4}{3}\pi R^3$$

Derivative formulas

$$D_{\mathbf{u}}f = \nabla f \cdot \mathbf{u}$$

Trig identities

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta) \qquad \cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$

$$\sin^2(\theta) = \frac{1}{2}(1-\cos(2\theta)) \qquad \cos^2(\theta) = \frac{1}{2}(1+\cos(2\theta))$$

Change of variables

$$G : \mathcal{D}_0 \to \mathcal{D}$$

$$G(u, v) = (x(u, v), y(u, v))$$

$$\mathrm{Jac}(G) = \frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial x}{\partial v}\frac{\partial y}{\partial u}$$

$$\iint_{\mathcal{D}} f(x, y)\,\mathrm{d}x\,\mathrm{d}y = \iint_{\mathcal{D}_0} f(x(u, v), y(u, v))\ |\mathrm{Jac}(G)|\ \mathrm{d}u\,\mathrm{d}v$$

Line integrals

$$\mathbf{r}(t) \text{ for } a \le t \le b \text{ parametrizing } \mathcal{C}$$

$$\int_{\mathcal{C}} f(x, y, z)\,\mathrm{d}s = \int_a^b f(\mathbf{r}(t))\,\|\mathbf{r}'(t)\|\,\mathrm{d}t$$

$$\int_{\mathcal{C}} \mathbf{F}\cdot\mathrm{d}\mathbf{r} = \int_a^b \mathbf{F}(\mathbf{r}(t))\cdot\mathbf{r}'(t)\,\mathrm{d}t$$

Surface integrals

$$G(u, v) = (x(u, v), y(u, v), z(u, v)) \text{ for } (u, v) \in \mathcal{D} \text{ parametrizing } \mathcal{S}$$

$$\mathbf{n}(u, v) = \mathbf{T}_u \times \mathbf{T}_v$$

$$\iint_{\mathcal{S}} f(x, y, z)\,\mathrm{d}S = \iint_{\mathcal{D}} f(G(u, v))\,\|\mathbf{n}(u, v)\|\,\mathrm{d}u\,\mathrm{d}v$$

$$\iint_{\mathcal{S}} \mathbf{F}\cdot\mathrm{d}\mathbf{S} = \iint_{\mathcal{D}} \mathbf{F}(G(u, v))\cdot\mathbf{n}(u, v)\,\mathrm{d}u\,\mathrm{d}v$$