Instructions: Wait to open the exam until instructed to do so. Then answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page. You will have 1 hour to complete this exam.

Question	Points	Score
1	30	
2	40	
3	10	
4	10	
5	10	
Total:	100	

Name: _____

Recitation Instructor: _____

Recitation Time: _____

1. For the following questions, suppose $\mathbf{u} = \langle 1, -1, 0 \rangle$ and $\mathbf{v} = \langle 0, -1, 1 \rangle$.

   (a) (5 points) Evaluate $\mathbf{u} - 3\mathbf{v}$.

   (b) (5 points) Evaluate $\mathbf{u} \cdot \mathbf{v}$.

   (c) (5 points) Find the angle between $\mathbf{u}$ and $\mathbf{v}$.

(d) (5 points) Evaluate $\mathbf{u} \times \mathbf{v}$.

(e) (5 points) Find the volume of the parallelepiped spanned by $\mathbf{u}$, $\mathbf{v}$ and the vector $\langle 3, 1, -2 \rangle$.

(f) (5 points) Find the distance between the point $Q = (1, 0, 0)$ and the line with direction vector $\mathbf{v}$ which passes through the point $P = (0, 1, 0)$.

2. Solve the problems regarding the points $P = (-2, 0, 1)$, $Q = (0, 1, 1)$ and $R = (-1, 1, 0)$.

   (a) (10 points) Find a normal vector to the plane containing P, Q and R.

   (b) (10 points) Write an equation for the plane containing P, Q and R.

(c) (10 points) Write an equation for a line passing through Q and perpendicular to the plane found in part (b).

(d) (10 points) Suppose S is any point on the line found in part (c). Find the vector projection of $\vec{PS}$ onto $\vec{PQ}$. Explain your response.

3. Sketch and describe the indicated traces of the quadric surface

$$x^2 + y + z^2 = 11$$

(a) (5 points) The $x = 1$ trace.

(b) (5 points) The $y = 2$ trace.

4. (10 points) Give the inequalities in Cartesian coordinates that describe the region below given in spherical coordinates

$$\rho \leq 3, \qquad \frac{\pi}{2} \leq \varphi \leq \pi, \qquad 0 \leq \theta \leq \pi$$

HINT: For the second inequality, consider the sign of $\cos\varphi$ and for the third, consider the sign of $\sin\theta$.

5. (10 points) If it exists, find

$$\lim_{t\to 0} \mathbf{r}(t)$$

for the vector valued function

$$\mathbf{r}(t) = \left\langle \frac{e^t - 1}{t}, \frac{t+1}{t^2+1}, \ln(t^2+1) \right\rangle.$$

Some formulas

$$\begin{aligned}\mathbf{u} &= \langle u_1, u_2, u_3 \rangle \\ \mathbf{v} &= \langle v_1, v_2, v_3 \rangle \\ \mathbf{u} \times \mathbf{v} &= \langle u_2v_3 - v_2u_3, u_3v_1 - u_1v_3, u_1v_2 - u_2v_1 \rangle\end{aligned}$$

$$\text{proj}_{\mathbf{u}}\mathbf{v} = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|^2}\mathbf{u}$$

Coordinate systems

Cylindrical

$$\begin{aligned}x &= r\cos(\theta) \\ y &= r\sin(\theta) \\ z &= z\end{aligned}$$

$$\begin{aligned}r &= \sqrt{x^2 + y^2} \\ \tan(\theta) &= \frac{y}{x} \\ z &= z\end{aligned}$$

Spherical

$$\begin{aligned}x &= \rho\cos(\theta)\sin(\varphi) \\ y &= \rho\sin(\theta)\sin(\varphi) \\ z &= \rho\cos(\varphi)\end{aligned}$$

$$\begin{aligned}\rho &= \sqrt{x^2 + y^2 + z^2} \\ \tan(\theta) &= \frac{y}{x} \\ \cos(\varphi) &= \frac{z}{\rho}\end{aligned}$$