Instructions: Wait to open the exam until instructed to do so. Then answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page. You will have 1 hour to complete this exam.

Question	Points	Score
1	10	
2	20	
3	15	
4	15	
5	10	
6	15	
7	15	
Total:	100	

Name: ______________________________

Recitation Instructor: ______________________________

Recitation Time: ______________________________

1. Answer the following questions concerning the vector valued function

$$\mathbf{r}(t) = \left\langle 4t^3, \pi\cos(\pi t), \frac{1}{t+1} \right\rangle$$

(a) (5 points) Evaluate $\mathbf{r}'(t)$.

(b) (5 points) Evaluate

$$\int_0^2 \mathbf{r}(t)\, \mathrm{d}t.$$

2. Consider the vector valued function

$$\mathbf{r}(t) = \langle e^t, 1 - 2e^t, 2e^t + 1 \rangle$$

for $0 \leq t \leq \ln(2)$.

(a) (10 points) Find the arc-length function $s(t)$ of $\mathbf{r}(t)$.

(b) (5 points) Find the arc-length parametrization $\mathbf{r}(s)$.

(c) (5 points) Recalling that curvature is $\kappa = \|\mathbf{r}''(s)\|$, find the curvature of the curve parametrized by $\mathbf{r}(t)$. Explain your answer by identifying the curve.

3. Find the limit, if it exists. If the limit does not exist, explain why.

(a) (5 points)

$$\lim_{(x,y)\to(0,0)} \frac{x^2y - xy}{x^4 + y^2x^2}$$

(b) (5 points)

$$\lim_{(x,y)\to(\pi/4,\pi/4)} \frac{\cos(x) - \sin(y)}{\cos^2(x) - \sin^2(y)}$$

(c) (5 points)

$$\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^4 + x^2y^2 + y^4}$$

4. Evaluate the partial derivatives, if they exist. If they do not exist, explain why.

   (a) (5 points) $f_y(3,6)$ for $f(x,y) = xe^{y-6} + \ln(xy)$.

   (b) (5 points) $\frac{\partial^3 f}{\partial x \partial y \partial z}(1,2,3)$ for $f(x,y,z) = xy + yz + xz + xyz$

   (c) (5 points) $f_z(0,3,0)$ for

$$f(x,y,z) = y + \sqrt{xyz + x^2 + z^2}$$

5. Consider the function $f(x, y) = xy - 2x + y^2$.

   (a) (5 points) Give the linearization $L(x, y)$ of $f(x, y)$ at $(-1, 1)$.

   (b) (5 points) Use your result from part (a) to write the equation of the tangent plane to the graph of $f(x, y)$ at $(-1, 1)$.

6. Let

$$f(x,y,z) = xy + yz + xz + 1$$

(a) (5 points) Find the gradient of f.

(b) (5 points) Let $\mathbf{u} = \left\langle \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3} \right\rangle$ and find the directional derivative $D_{\mathbf{u}} f\left(\sqrt{3}, \sqrt{3}, \sqrt{3}\right)$.

(c) (5 points) Give an example of a vector $\mathbf{v}$ for which f is increasing in the direction of $\mathbf{v}$ starting at $(-1, 1, 1)$.

7. Consider the function
$$f(x, y, z) = x^2 + y^2 + z$$

(a) (10 points) Use the chain rule to calculate $\frac{\partial f}{\partial \varphi}$ at the point $(\rho, \theta, \varphi) = (1, 0, \pi/2)$ in spherical coordinates.

(b) (5 points) Confirm your result by expressing f in spherical coordinates and taking its partial derivative with respect to φ.

Coordinate systems

Cylindrical

$$x = r\cos(\theta)$$
$$y = r\sin(\theta)$$
$$z = z$$

$$r = \sqrt{x^2+y^2}$$
$$\tan(\theta) = \frac{y}{x}$$
$$z = z$$

Spherical

$$x = \rho\cos(\theta)\sin(\varphi)$$
$$y = \rho\sin(\theta)\sin(\varphi)$$
$$z = \rho\cos(\varphi)$$

$$\rho = \sqrt{x^2+y^2+z^2}$$
$$\tan(\theta) = \frac{y}{x}$$
$$\cos(\varphi) = \frac{z}{\rho}$$

Derivative formulas

Directional derivative : $D_{\mathbf{u}}f(x,y) = \nabla f(x,y)\cdot\mathbf{u}$

Chain rule, general case for $f(x_1,\ldots,x_m)$ and $x_j(t_1,\ldots,t_n)$:

$$\frac{\partial f}{\partial t_i} = \frac{\partial f}{\partial x_1}\frac{\partial x_1}{\partial t_i} + \cdots + \frac{\partial f}{\partial x_m}\frac{\partial x_m}{\partial t_i}$$

Trigonometric values

$$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right),$$
$$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} = \cos\left(\frac{\pi}{6}\right),$$
$$\sin(0) = 0 = \cos\left(\frac{\pi}{2}\right)$$
$$\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} = \cos\left(\frac{\pi}{4}\right),$$
$$\sin\left(\frac{\pi}{2}\right) = 1 = \cos(0)$$