Instructions: Wait to open the exam until instructed to do so. Then answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page. You will have 1 hour and 50 minutes to complete this exam.

Question	Points	Score
1	15	
2	20	
3	15	
4	10	
5	20	
6	20	
7	20	
Total:	120	

Name: ______________________________

Recitation Instructor: ______________________________

Recitation Time: ______________________________

1. Let $\mathbf{v} = \langle 2, 1, 1 \rangle$ and $\mathbf{w} = \langle -1, 0, -1 \rangle$.

   (a) (5 points) Compute the area of the parallelogram spanned by $\mathbf{v}$ and $\mathbf{w}$.

   (b) (5 points) Is the angle θ between $\mathbf{v}$ and $\mathbf{w}$ acute, obtuse or a right angle? Explain your response.

   (c) (5 points) Give an equation for the line passing through $(2, 1, -2)$ to with direction vector $\mathbf{v}$.

2. Calculate the following quantities if they exist. Otherwise, explain why they do not exist. Justify either response.

   (a) (5 points)

$$\lim_{(x,y)\to(0,0)} \frac{xy - y^2}{3x^2 + 2y^2}$$

   (b) (5 points) For

$$f(x, y, z) = xe^{xy} - \cos(yz)$$

   compute

$$f_{yz}(x, y, z)$$

(c) (5 points) For $f(x, y) = x^2 + y^2 - xy$ find the unit vector which points in the direction for which $f(x, y)$ increases the most rapidly starting at $(2, 1)$.

(d) (5 points) Find the equation for the tangent plane to the surface

$$z = x^2 + y^2 - xy$$

at the point $(2, 1, 3)$.

3. Let

$$f(x,y) = x^2 + 2xy - 2y$$

and $\mathcal{D}$ be the triangle in the fourth quadrant with bounds

$$x \geq 0, \qquad y \leq 0, \qquad y - x \geq -4$$

.

(a) (5 points) Find the critical points of $f(x,y)$ in the interior of $\mathcal{D}$.

(b) (5 points) Does $f(x,y)$ have a local max, local min or saddle point at the point(s) found in (a)? Explain your response.

(c) (5 points) Find the maximum value of $f(x, y)$ on $\mathcal{D}$.

4. (10 points) Let $\mathcal{E} = [1,2] \times [-2,1] \times [0,3]$. Evaluate the triple integral

$$\iiint_{\mathcal{E}} 3z^2 - 4xy \; \mathrm{d}V$$

5. Evaluate the following integrals.

   (a) (10 points) Let $\mathcal{D}$ be the region $x^2 + y^2 \leq 9$ and $y \leq 0$. Evaluate

$$\iint_{\mathcal{D}} 2e^{x^2+y^2} \, \mathrm{d}A.$$

(b) (10 points) Let $\mathcal{D}$ be the region

$$0 \le x \le \frac{\pi}{2}, \qquad 0 \le y \le \sin x.$$

Compute the integral

$$\iint_{\mathcal{D}} 2y \cos x \ \mathrm{d}A.$$

6. Let

$$\mathbf{F} = \langle -2x, -2y, 4z \rangle .$$

(a) (5 points) If $\mathbf{F}$ is a conservative vector field, find a potential. Otherwise, explain why it is not conservative.

(b) (5 points) Let C be the oriented curve with parametrization

$$\mathbf{r}(t) = \left\langle \cos(t^2 - t), e^{\sin(\pi t)}, t - 1 \right\rangle$$

for $0 \leq t \leq 1$. Compute

$$\int_C \mathbf{F} \cdot d\mathbf{r}.$$

(c) (5 points) Let $\mathbf{A} = \langle -2yz, 2xz, 0\rangle$ and compute curl($\mathbf{A}$).

(d) (5 points) Let $\mathcal{S}$ be the upper ellipsoid

$$\frac{x^2}{4} + y^2 + \frac{z^2}{9} = 1, \qquad z \geq 0$$

oriented outwardly. Compute the surface integral

$$\iint_{\mathcal{S}} \mathbf{F} \cdot d\mathbf{S}.$$

State any theorems used in the computation.

7. Let $\mathcal{S}$ be the cone

$$x^2 + y^2 = z^2, \qquad -2 \leq z \leq 0$$

and $\mathcal{D}$ the disc

$$x^2 + y^2 \leq 4, \qquad z = -2$$

both oriented outwardly from the interior and $\mathbf{F} = \langle x, y, z \rangle$.

(a) (5 points) Compute $\operatorname{div} \mathbf{F}$.

(b) (5 points) Compute

$$\iint_{\mathcal{D}} \mathbf{F} \cdot \, d\mathbf{S}$$

(c) (5 points) The volume of a cone is $\frac{1}{3}Ah$ where h is the height of the cone and A is the area of the base. Using this and the Divergence Theorem, calculate

$$\iint_{\mathcal{S}} \mathbf{F} \cdot \, d\mathbf{S}$$

(d) (5 points) Give another explanation of the result in (c) by calculating $\mathbf{F} \cdot \mathbf{N}$ where $\mathbf{N}$ is the orientation vector field on $\mathcal{S}$.

Coordinate systems

Polar	Cylindrical	Spherical
$x = r\cos(\theta)$	$x = r\cos(\theta)$	$x = \rho\cos(\theta)\sin(\phi)$
$y = r\sin(\theta)$	$y = r\sin(\theta)$	$y = \rho\sin(\theta)\sin(\phi)$
	$z = z$	$z = \rho\cos(\phi)$
$r = \sqrt{x^2+y^2}$	$r = \sqrt{x^2+y^2}$	$\rho = \sqrt{x^2+y^2+z^2}$
$\tan(\theta) = \dfrac{y}{x}$	$\tan(\theta) = \dfrac{y}{x}$	$\tan(\theta) = \dfrac{y}{x}$
	$z = z$	$\cot(\phi) = \dfrac{z}{\sqrt{x^2+y^2}}$
$\mathrm{d}x\,\mathrm{d}y = r\,\mathrm{d}r\,\mathrm{d}\theta$	$\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = r\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}z$	$\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = \rho^2\sin\phi\,\mathrm{d}\rho\,\mathrm{d}\phi\,\mathrm{d}\theta$

Unit vectors

$$\mathbf{T} = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|} \qquad \mathbf{N} = \frac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$$

Useful area and volume formulas

$$\text{Surface area of sphere of radius } R = 4\pi R^2$$
$$\text{Volume of sphere of radius } R = \frac{4}{3}\pi R^3$$

Derivative formulas

$$\text{Directional derivative}: D_{\mathbf{u}}f(x,y) = \nabla f(x,y)\cdot\mathbf{u},$$
$$\text{Discriminant}: D = f_{xx}(x_0,y_0)f_{yy}(x_0,y_0) - f_{xy}(x_0,y_0)^2$$

Trig identities

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta) \qquad \cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$

$$\sin^2(\theta) = \frac{1}{2}(1-\cos(2\theta)) \qquad \cos^2(\theta) = \frac{1}{2}(1+\cos(2\theta))$$

Change of variables

$$G : \mathcal{D}_0 \to \mathcal{D}$$

$$G(u,v) = (x(u,v), y(u,v))$$

$$\mathrm{Jac}(G) = \frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial x}{\partial v}\frac{\partial y}{\partial u}$$

$$\iint_{\mathcal{D}} f(x,y)\,\mathrm{d}x\,\mathrm{d}y = \iint_{\mathcal{D}_0} f(x(u,v), y(u,v))\ |\mathrm{Jac}(G)|\ \mathrm{d}u\,\mathrm{d}v$$

Line integrals

$$\mathbf{r}(t) \text{ for } a \le t \le b \text{ parametrizing } \mathcal{C}$$

$$\int_{\mathcal{C}} f(x,y,z)\,\mathrm{d}s = \int_a^b f(\mathbf{r}(t))\,\|\mathbf{r}'(t)\|\,\mathrm{d}t$$

$$\int_{\mathcal{C}} \mathbf{F}\cdot\mathrm{d}\mathbf{r} = \int_a^b \mathbf{F}(\mathbf{r}(t))\cdot\mathbf{r}'(t)\,\mathrm{d}t$$

Surface integrals

$$G(u,v) = (x(u,v), y(u,v), z(u,v)) \text{ for } (u,v)\in\mathcal{D} \text{ parametrizing } \mathcal{S}$$

$$\mathbf{n}(u,v) = \mathbf{t}_u \times \mathbf{t}_v$$

$$\iint_{\mathcal{S}} f(x,y,z)\,\mathrm{d}S = \iint_{\mathcal{D}} f(G(u,v))\,\|\mathbf{n}(u,v)\|\,\mathrm{d}u\,\mathrm{d}v$$

$$\iint_{\mathcal{S}} \mathbf{F}\cdot\mathrm{d}\mathbf{S} = \iint_{\mathcal{D}} \mathbf{F}(G(u,v))\cdot\mathbf{n}(u,v)\,\mathrm{d}u\,\mathrm{d}v$$