Test 1 Version A
Math 222 Fall 2022
September 15, 2022
Name: _____

Time of Recitation: _____

Initials of Recitation Instructor: _____

You may not use any type of calculator whatsoever. (Cell phones off and away!) You are not allowed to have any other notes, and the test is closed book. Use the backs of pages for scrapwork, and if you write anything on the back of a page which you want to be graded, then you should indicate that fact (on the front). Do not unstaple or remove pages from the exam. **Except for matters of English, the proctors will not answer any questions.**

By taking this exam you are agreeing to abide by KSU's Academic Integrity Policy.

Simple or standard simplifications should be made. You must **show your work**, and in order to get credit or partial credit, your work must make sense!

GOOD LUCK!!!

Problem	Possible	Score	Problem	Possible	Score
1	16		5	12	
2	16		6	12	
3	16		7	16	
4	12				
Total	60			40	

1. Define $\vec{u} := (-2, 3, 1)$ and $\vec{w} := (5, -3, -4)$. Compute the following:

   (a) $|\vec{u}|$.

   (b) $\vec{u} \bullet \vec{w}$.

   (c) $\vec{u} \times \vec{w}$.

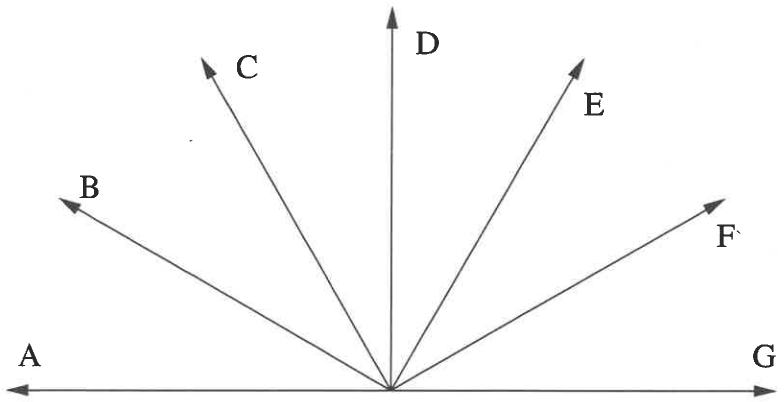
   (d) The area of the triangle with two sides given by $\vec{u}$ and $\vec{w}$.

2. Find an equation for the plane which contains the points

$$(1, 1, 1),\ (3, 0, 1),\ (2, 0, 2).$$

   Express your answer in the form: $Ax + By + Cz = D$.

3. For this problem we refer to the following figure which is drawn to scale:



All of the vectors in the figure have length **four**. All of the angles between the vectors are multiples of $\pi/6$ radians. (So they are multiples of 30 degrees.) Compute the following explicitly:

(a) $G \bullet A$.

(b) $||C \times F||$.

(c) $||B \times B||$.

(d) $E \bullet D$.

(e) $||E - B||$.

(f) $|C \times G| - |E \times G|$.

(g) $\dfrac{C \bullet B}{F \bullet G}$.

4. For the curve $\vec{r}(t) = (3t,\ \cos(4t),\ \sin(4t))$, find the unit tangent, the unit normal, the unit binormal, the curvature, and the tangential and the normal acceleration.

5. Match ups. Here are some equations...

(a) $2x - 3y - z = 0$

(b) $x^2 + y^2 - z^2 = 4$

(c) $x = y^2$

(d) $z^2 - x^2 - y^2 = 0$

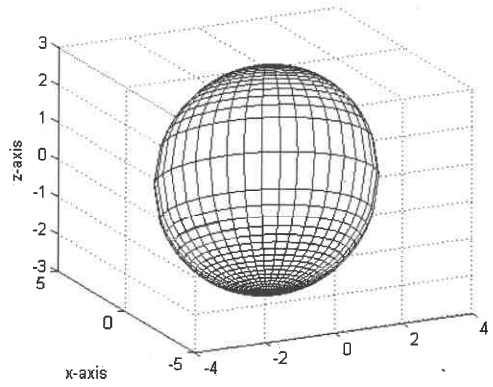
(e) $x^2 + y^2 + z^2 = 9$

(f) $z^2 - x^2 - y^2 = 4$

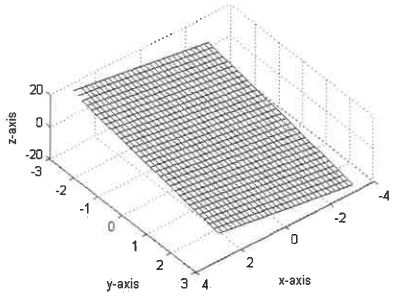
(g) $x^2 + y^2 - z^2 = 0$

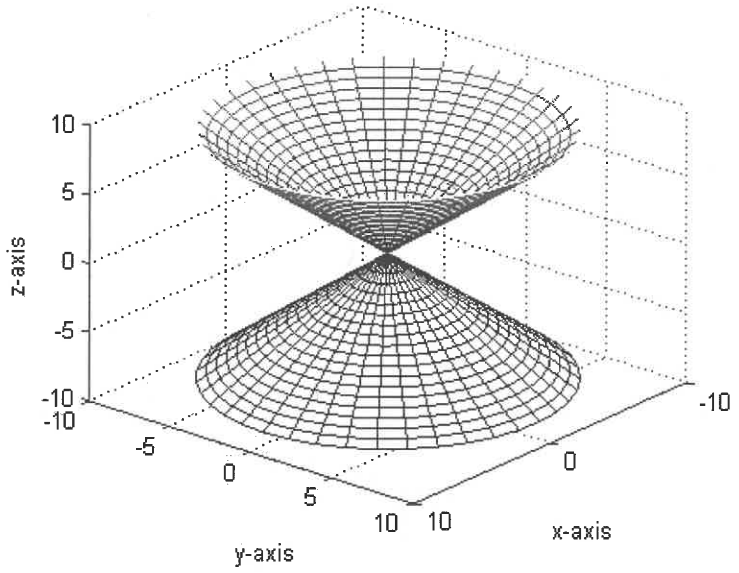
For each of the next four surfaces, determine which equation above determines it.

A

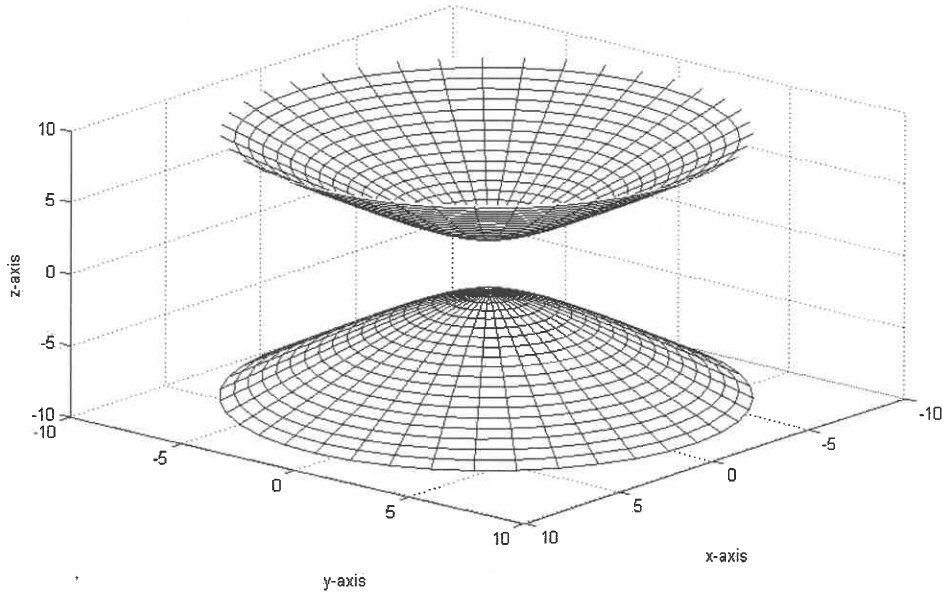


B




10
5
0
-5
-10
z-axis
-10
-5
0
5
10
y-axis
10
0
-10
x-axis

C


10
5
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-5
-10
z-axis
-10
-5
0
5
10
y-axis
10
5
0
-5
-10
x-axis

D

6. Write down a vector normal to the plane

$$4x - 3y + 2z = 5\,,$$

and then find the distance from the point $(-1, -2, -3)$ to this plane.

7. Short answers and multiple choice...

(a) Which has larger curvature, a circle with radius 5 or a circle with radius 25?

(b) What is the curvature of a circle with radius 49?

(c) The dot product of your velocity vector with your acceleration vector will be the most negative when

i. You are driving 40 miles per hour around a curve, and you are not pressing down on the gas or the brake.

ii. You are driving 40 miles per hour and you are pressing down on the gas pedal to speed up to 70 miles per hour.

iii. You are driving 50 miles per hour, but you suddenly see something ahead of you which causes you to step on the brake hard.

iv. Your car is parked.

(d) Label the following as reasonable (R) or unreasonable (U).

i. $3/\vec{v}$

ii. $\vec{v}/3$

iii. $\vec{u} \times (\vec{v} \bullet \vec{w})$

iv. $\vec{u} \times (\vec{v} \times \vec{w})$

Curves:

$$\vec{r}(t) = (x(t), y(t), z(t))$$

$$\text{Arclength} \quad s(t) := \int_a^t ||\vec{r}\,'(u)||\ du \quad \text{(measuring from } a \text{ to } t\text{)}$$

$$\text{Unit Tangent Vector} \quad \vec{T}(t) := \frac{\vec{r}\,'(t)}{||\vec{r}\,'(t)||}$$

$$\text{Curvature Vector} \quad \frac{d\vec{T}}{ds} = \frac{\vec{T}\,'(t)}{||\vec{r}\,'(t)||}$$

$$\text{Curvature} \quad \kappa := \left|\left|\frac{d\vec{T}}{ds}\right|\right|$$

$$\text{Principle Unit Normal} \quad \vec{N}(t) := \frac{1}{\kappa}\frac{d\vec{T}}{ds} = \frac{\vec{T}\,'(t)}{\left|\left|\vec{T}\,'(t)\right|\right|}$$

$$\text{Binormal} \quad \vec{B}(t) := \vec{T}(t) \times \vec{N}(t)$$

$$\vec{B}(t) = \frac{\vec{r}\,' \times \vec{r}\,''}{||\vec{r}\,' \times \vec{r}\,''||}$$

$$\vec{a}(t) = \frac{d^2s}{dt^2}\vec{T} + \left(\frac{ds}{dt}\right)^2 \kappa\vec{N} = a_T\vec{T} + a_N\vec{N}$$

$$a_T = \frac{\vec{r}\,' \bullet \vec{r}\,''}{||\vec{r}\,'||} \qquad a_N = \frac{||\vec{r}\,' \times \vec{r}\,''||}{||\vec{r}\,'||} \qquad \kappa = \frac{||\vec{r}\,' \times \vec{r}\,''||}{||\vec{r}\,'||^3}$$

For the curve given by the graph $y = f(x)$ we have:

$$\kappa(x) = \frac{|f''(x)|}{(1 + f'(x)^2)^{3/2}}\ .$$

For the plane curve given by $\vec{r}(t) = (x(t), y(t))$ we have:

$$\kappa(t) = \frac{|x'(t)y''(t) - x''(t)y'(t)|}{(x'(t)^2 + y'(t)^2)^{3/2}}\ .$$