1. (10 points) §2.1: Let $P = (0, 2)$, $Q = (1, 3)$, $R = (-4, -6)$.

   (a) Find and express in both component form and by using standard unit vectors:

   $$\overrightarrow{PQ},\ \overrightarrow{PR},\ \overrightarrow{PQ} - \overrightarrow{PR}$$

   (b) Find a unit vector in direction $\overrightarrow{PR}$.

   (c) Find $\mathbf{v}$ with the same direction as $\overrightarrow{PQ} - \overrightarrow{PR}$ and length 10.

2. (10 points) §2.2:

   (a) Give an equation for S, the set of points of distance 5 from the point $C = (2, -4, -6)$ in $\mathbb{R}^3$.

   (b) Give an equation to describe the intersection of S and the plane $y = -3$.

   (c) Given $P = (2, -9, -6)$ in S, find the unique point Q of distance 10 from P which is also in S.

   (d) Find a unit-length vector in the direction of $\overrightarrow{PQ}$.

3. (10 points) §2.3: Let $\mathbf{u} = \langle 1, 4, -3 \rangle$ and $\mathbf{v} = \langle 2, -5, 2 \rangle$.

   (a) Find $\mathbf{u} \cdot \mathbf{v}$. Find the angle $\theta \in [0, \pi]$ between $\mathbf{u}, \mathbf{v}$.

   (b) Find $\left\| \text{proj}_{\mathbf{u}}(\mathbf{v}) \right\|$. Find $\text{proj}_{\mathbf{u}}(\mathbf{v})$

(c) Are $\mathbf{v}$ and the standard basis vector $\mathbf{k}$ orthogonal? Why or why not?

(d) Find the work done by a force $\mathbf{F} = \langle 1, -4, -3 \rangle$ applied to move an object in a straight line from the terminal point of $\mathbf{u}$ to the terminal point of $\mathbf{v}$. Let units be in feet and lbs.

4. (10 points) §2.4: Let $\mathbf{u} = \langle 4, 1, 3 \rangle$ and $\mathbf{v} = \langle -5, 1, 6 \rangle$.

   (a) Find $\mathbf{u} \times \mathbf{v}$ and $\|\mathbf{u} \times \mathbf{v}\|$.

   (b) Find a unit-length vector orthogonal to both $\mathbf{u}$ and $\mathbf{v}$.

   (c) Let $P = \langle 2, -5, 7 \rangle$. Find the area of the triangle spanned by P and the terminal points of $\mathbf{u}$, $\mathbf{v}$

(d) Find the volume of the parallelopiped spanned by $\mathbf{u}, \mathbf{v}$ and $-\mathbf{j}$, where $\mathbf{j} = \langle 0, 1, 0 \rangle$.

(e) Are $\mathbf{u}$ and $\langle -3, 4, -1 \rangle$ orthogonal? Why or why not?

5. (10 points) §2.5: Let $P = (1, 1, -1)$, $Q = (-7, -6, 4)$, and $R = (-3, 2, 8)$.

   (a) Describe the line $\mathcal{L}$ through P and Q in vector, parametric, and symmetric form.

   (b) Find the distance from R to the line $\mathcal{L}$ through P and Q.

(c) Describe the plane Π through P, Q and R in vector, scalar, <u>or</u> general form (any form is acceptable).

(d) Classify the relationship between the line $\mathcal{L}$ through P, Q and the line $\mathcal{M}$ through R and $(-10, -5, 2)$. Are $\mathcal{L}$ and $\mathcal{M}$: equal, parallel but not equal, intersect but not equal, or skew?

(e) Find the distance from $(2, -2, 2)$ to the plane Π.

6. (10 points) §2.6:

   (a) Classify the following quadric surface and identify the axis of the surface:

$$49x^2 - 392x + 16y^2 - 32y - 784z + 2336 = 0$$

   (b) Give the trace in the $z = 5$ plane.

7. (10 points) §2.7:

   (a) Identify the following surfaces in spherical coordinates, and sketch a graph of the surface:

      i. $\phi = \frac{\pi}{2}$.

      ii. $\rho = \cos\theta \sin\phi$. Hint: Multiply both sides by ρ.

   (b) Convert the following points from rectangular to both cylindrical and spherical coordinates, respectively:

      i. $(3, 1, 5)$.

      ii. $(-2, 1, 7)$.

8. (10 points) §3.1: Let $\mathbf{r}(t) = \left\langle e^{-4t}, e^{\frac{1}{2t}}, \ln(2t) - 7 \right\rangle$.

(a) Find $\lim_{t\to 10} \mathbf{r}(t)$.

(b) Is $\mathbf{r}(t)$ continuous at $t = 10$?

(c) Are there any domain restrictions on $\mathbf{r}(t)$?

(d) Are there any values for t at which $\mathbf{r}(t)$ is not continuous?

(e) Let C be the space curve given by $\mathbf{r}(t) = \langle t, t^3, 20 \rangle$.
Describe the curve: What shape is it?

(f) C is contained in a unique plane in $\mathbb{R}^3$. Which plane is C contained in?

9. (10 points) §3.2:

(a) Using the limit definition of the derivative, find $\mathbf{r}'(t)$ for

$$\mathbf{r}(t) = \langle t^2, -4t \rangle.$$

(b) Using your formula from part (a), find $\mathbf{r}'(5)$.

(c) Let $\mathbf{u}(t) = \left\langle t, t^2, t^3 \right\rangle$ and $\mathbf{w}(t) = \left\langle 2t + 3, \ln(t), 12 \right\rangle$. Find

$$\int \mathbf{u}(t) \times \mathbf{w}(t)\, \mathrm{d}t$$

(d) Find $\frac{d}{dt}[\mathbf{u}(t) \times \mathbf{w}(t)]$

10. (10 points) §3.3: Let $\mathbf{r}(t) = \langle 3\cos(2t), 3\sin(2t), 12t \rangle$.

(a) Find the arc length of $\mathbf{r}(t)$ over $t \in [0, 4\pi]$.

(b) Give an arc length parametrization of $\mathbf{r}(t)$ for $t > 0$.

(c) Find the principal unit normal vector $\mathbf{N}(t)$ at $t = 4\pi$.