

Final Test

Math 240 Summer 2013

August 2<sup>nd</sup>, 2013

Name: \_\_\_\_\_

Name of Recitation Instructor: \_\_\_\_\_

By taking this exam you are agreeing to abide by KSU's Academic Integrity Policy.

If you get stuck on any problem, try other problems. Time is limited...

Show your work to get credit or partial credit, your work must make sense. Answers without steps will NOT give you full credit. No calculators.

Page	Possible	Score
1	15	
2	15	
3	15	
4	15	
5	15	
6	15	
7	10	

1 (15 points) Solve the linear system

$$\begin{cases} \frac{dx}{dt} = 2x + y + \delta(t), & x(0) = 0 \\ \frac{dy}{dt} = -x + 2y, & y(0) = 2 \end{cases}$$

2 (15 points) Solve

$$(x+1)y'' - 4xy = 0$$

$$y(0) = 1, y'(0) = 0$$

using a power series. Find the recurrence relation and the first THREE non-zero terms of the series solution.

3 (15 points) Find the roots of the indicial equation about  $x_0 = 0$  for

$$(x^3 - x^2)y'' + (x^2 + 4x)y' + (x - 4)y = 0.$$

4 (15 points) Find and classify the equilibria for:

$$\begin{cases} \frac{dx}{dt} = x - xy \\ \frac{dy}{dt} = 3x + 3y \end{cases}$$

5 (15 points) Find the general solution to the following Euler equation:

$$x^2 y'' - \frac{3}{2} x y' + \frac{3}{2} y = 0.$$

6 (15 points) Assuming that  $a \geq 0$ , find  $a$  so that there are infinitely many solutions to the boundary value problem:

$$y'' - 2y' + (1 + a^2)y = 0,$$

$$y(0) = 0, y(\pi) = 0.$$

7. (10 points) Match the equations to the poles of the Laplace transform of the solution. Each picture is the complex plane with real part  $[-10, 1]$  and imaginary part  $[-10i, 10i]$ . The poles are marked with "X", and double poles are marked with circled "X".

A.  $x''+9x=0, x(0)=10, x'(0)=0;$

B.  $x''+2x'+9x=0, x(0)=10, x'(0)=0;$

C.  $x''+8x'+9x=0, x(0)=10, x'(0)=0;$

D.  $x''+6x'+9x=0, x(0)=10, x'(0)=0;$

E.  $x''-x'+9x=0, x(0)=10, x'(0)=0.$

