Name:____________________________________

Recitation:____________________________________

Math 240
Exam 3
November 18, 2014

Problem	Score
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Closed book. You may use a calculator and one 8 ½ × 11" sheet of handwritten notes (both sides). You must show your work to receive full credit. Write solutions in explicit form if possible. All problems have a solution that can be found using the techniques of this class. **Series solutions should be listed at least through the x^4 term unless otherwise specified.**

Pledge:
On my honor, as a student, I have neither given nor received unauthorized aid on this examination: ______________________________________ ____________

(signature) (date)

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1. Note: you may use the table of Laplace transforms which is attached at the end of the test.

a) Find the Laplace transform of $f(t) = 2\cos(t) + 1$

b) Find the inverse Laplace transform of $F(s) = \dfrac{2s+2}{s^2+6s+13}$.

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2. Solve the initial value problem

$$y''+8y'+15y=\delta(t), \quad y(0)=0, \quad y'(0)=0.$$

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3. Solve the initial value problem,

$$y'' + (x+4)y = 0, \quad y(0) = 1, \; y'(0) = -4.$$

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4. Solve the initial value problem (your solution will involve an integral)

$$y'' + 4y' + 5y = f(t), \quad y(0) = 0, \quad y'(0) = 0$$

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5. Find and classify the equilibria for the system,

$$\frac{dx}{dt} = 3x^2 + 4y^2 - 16,$$
$$\frac{dy}{dt} = x - 2y$$

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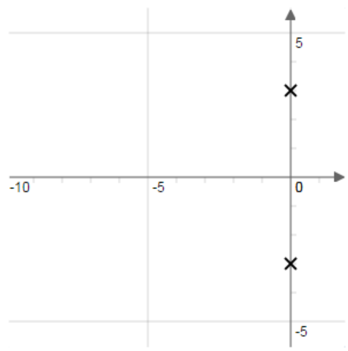
6. Match the poles of the Laplace transform on the left with the graphs of the functions on the right.



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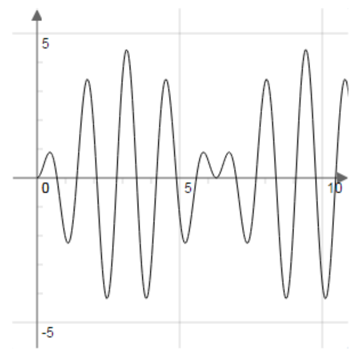
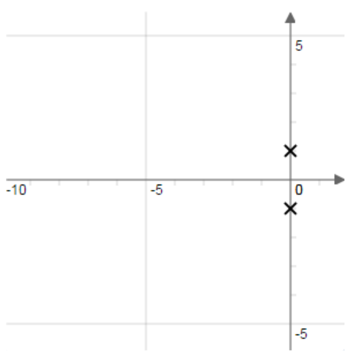
(A)





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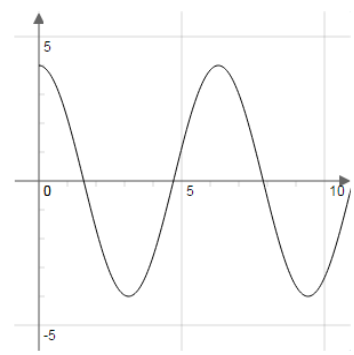
(B)



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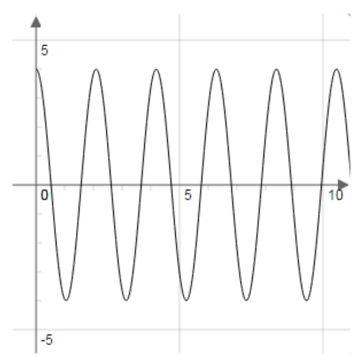

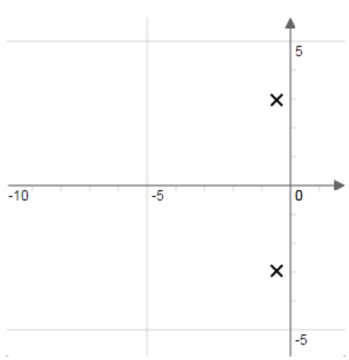

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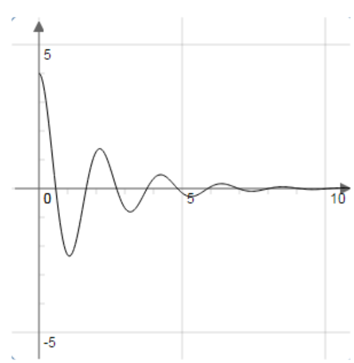
(D)



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7. The recurrence relation for the series solution $y = \sum_{n=0}^{\infty} a_n x^n$ for the equation $y'' - xy' + 2y = 0$ is $a_{n+2} = \frac{n-2}{(n+2)(n+1)} a_n$ for $n \geq 0$. If you are given $y''(0) = 4$, what is the value of $y(0)$?

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8. Suppose x and y satisfy the system
$$\begin{aligned} \frac{dx}{dt} &= 10x - 2xy, \quad x(0) = 4 \\ \frac{dy}{dt} &= 3y - xy, \quad\quad y(0) = 4 \end{aligned}.$$

Find $\lim_{t\to\infty} x(t)$ and $\lim_{t\to\infty} y(t)$.

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