MATH 240 SPRING 2014

EXAM 2

Problem 1. (4 pts)

Your name:

Recitation instructor name:

Recitation time:

Problem	1	2	3	4	5	6	7	Total
Grade								

Trigonometry reminder:

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin(x)$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos(x)$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0

Problem 2. (16 pts) Find and classify (as stable, semi-stable or unstable) the equilibria of the autonomous equation

$$\frac{dp}{dt} = p^3 - 4p^2 + 4p \quad \text{as} \quad t \to \infty.$$

If $p(t)$ is the solution of the equation, satisfying the initial condition $p(0) = 1$, what is the limit $\lim_{t\to\infty} p(t)$?

Problem 3. (16 pts) Consider the initial value problem

$$\frac{dy}{dx} = y^2, \quad y(0) = 1.$$

Find the Picard iterations $y_1(x)$ and $y_2(x)$ for this problem, starting with $y_0 = 1$.

Problem 4. (16 pts) Solve the initial value problem

$$y'' - 4y' + 4y = 2e^x, \quad y(0) = y'(0) = 0.$$

Problem 5. (16 pts) Find the general solution of the equation

$$y'' + 2y' + 3y = 2\cos(x).$$

Problem 6. (16 pts) Solve the initial value problem

$$\ddot{x} + 3\dot{x} + 2x = 3e^{-t}, \quad x(0) = \dot{x}(0) = 0.$$

Problem 7. (16 pts) Suppose an undamped spring-mass system has a mass of 5 g and resonates at a frequency of 13 Hz ($13\frac{\text{cycles}}{\text{sec}}$). A damping mechanism is then attached to the system, and it is observed that the free damped motion of the system is quasi-periodic with a frequency of 12 Hz. What is the spring constant of the system? What is the damping constant of the attached mechanism?