MATH 240 SPRING 2014

EXAM 3

Problem 1. (4 pts)

Your name:

Recitation instructor name:

Recitation time:

Problem	1	2	3	4	5	6	7	Total
Grade								

Problem 2. (16 pts) Solve the following initial value problem, using Laplace transform:

$$\ddot{x} + 4\dot{x} + 5x = 0, \quad x(0) = 1, \quad \dot{x}(0) = 2.$$

Problem 3. (16 pts) Solve the following initial value problem, using Laplace transform:

$$\ddot{x} + 4\dot{x} + 3x = \delta(t-1), \quad x(0) = \dot{x}(0) = 0.$$

Problem 4. (16 pts) Solve as a convolution integral (do not evaluate the integral)

$$\ddot{x} - 2\dot{x} + x = e^{t^2}, \quad x(0) = \dot{x}(0) = 0.$$

Problem 5. (16 pts) Find and classify (as stable, unstable or saddle) the equilibria of the system

$$\begin{cases} \dot{x} = x^2 - xy \\ \dot{y} = x + y + 2 \end{cases}$$

Problem 6. (16 pts) As we have seen, the general solution of the homogeneous Euler equation

$$x^2y'' + xy' - y = 0$$

is

$$y = Ax + \frac{B}{x},$$

where A and B are arbitrary constants. Use variation of parameters to solve the initial value problem

$$x^2y'' + xy' - y = 4x^3, \quad y(1) = y'(1) = 0.$$

Problem 7. (16 pts) Let

$$y(x) = \sum_{n=0}^{\infty} y_n x^n$$

be the solution of the initial value problem

$$(x^2 + 4)y'' + xy' - y = 0, \quad y(0) = 1, \quad y'(0) = 0.$$

Determine the recurrence relation for the coefficients y_n. Find the coefficients y_0, y_1, y_2, y_3 and a lower bound for the radius of convergence of the power series $y(x)$.

Laplace transform

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1	$\frac{1}{s}$
t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
$\sin(at)$	$\frac{a}{s^2+a^2}$
$\cos(at)$	$\frac{s}{s^2+a^2}$
$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$
$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
$\delta(t-c)$	e^{-cs}
$u(t-c)f(t-c)$	$e^{-cs}F(s)$
$f'(t)$	$sF(s) - f(0)$
$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$\int_0^t f(t-\tau)g(\tau)d\tau$	$\mathcal{L}\{f(t)\}\mathcal{L}\{g(t)\}$