MATH 240 SPRING 2014

FINAL EXAM

Your name:

Recitation instructor name:

Recitation time:

Problem	1	2	3	4	5	6	7	8	Total
Grade									

Problem 1. (25 pts) Solve the initial value problem

$$\frac{dy}{dx} + y = e^x y^2, \quad y(0) = 1.$$

Problem 2. (25 pts) Solve the initial value problem

$$\frac{dy}{dx} = \frac{8x - 3y}{3x - y}, \quad y(0) = 1.$$

Problem 3. (25 pts) Solve the initial value problem

$$\ddot{x} - 3\dot{x} + 2x = e^t, \quad x(0) = \dot{x}(0) = 0.$$

Problem 4. (25 pts) Find and classify (as stable, unstable or saddle) the equilibria of the system

$$\begin{cases} \dot{x} = xy - 1 \\ \dot{y} = x - y \end{cases}$$

Problem 5. (25 pts) Solve the initial value problem

$$x^2y'' + 5xy' + 4y = 0, \quad y(1) = 0, y'(1) = 1.$$

Problem 6. (25 pts) It is known that the general solution of the homogeneous Euler equation

$$x^2y'' - 2xy' + 2y = 0$$

is

$$y = Ax + Bx^2,$$

where A and B are arbitrary constants. Use variation of parameters to solve the initial value problem

$$x^2y'' - 2xy' + 2y = x, \quad y(1) = y'(1) = 0.$$

Problem 7. (25 pts) Let

$$y(x) = \sum_{n=0}^{\infty} y_n x^n$$

be the solution of the initial value problem

$$(x^2 + 1)y'' + xy' - 4y = 0, \quad y(0) = 1, \quad y'(0) = 0.$$

Determine the recurrence relation for the coefficients y_n. Find the coefficients y_0, y_1, y_2, y_3 and a lower bound for the radius of convergence of the power series $y(x)$.

Problem 8. (25 pts) Find and classify (as regular or irregular) the singular points of the equation

$$(x^3 - 2x^2 + x)y'' + (x - 1)y' - y = 0.$$

Solve the indicial equation for every regular singular point you found.

Laplace transform

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1	$\frac{1}{s}$
t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
$\sin(at)$	$\frac{a}{s^2+a^2}$
$\cos(at)$	$\frac{s}{s^2+a^2}$
$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$
$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
$\delta(t-c)$	e^{-cs}
$u(t-c)f(t-c)$	$e^{-cs}F(s)$
$f'(t)$	$sF(s) - f(0)$
$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$\int_0^t f(t-\tau)g(\tau)d\tau$	$\mathcal{L}\{f(t)\}\mathcal{L}\{g(t)\}$