Math 240 Summer 2014
Final Exam

Aug. 1, 2014

Student Name: _____

Instructor: _____

Class time: _____

	Score
1	
2	
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Total	

Problem 1. (18 points) Find and classify the equilibrium points for the following system of equations:

$$\begin{cases} x'(t) = x^2 - y \\ y'(t) = y - x - 2 \end{cases}$$

Problem 2. (20 points) Find all the solutions for

$$y' + 2xy = 2e^{x^2-2x}y^2.$$

Then solve the initial value problem:

$$y' + 2xy = 2e^{x^2-2x}y^2, \quad y(0) = 1$$

Problem 3. (16 points) Use Laplace transform to solve the initial value problem:

$$x''(t) + 4x(t) = e^{-t}, \quad x(0) = x'(0) = 0$$

Problem 4. (24 points) Use change of variable to solve the Euler equation:

$$x^2y''(x) + 7xy'(x) + 9y(x) = 0$$

Then use variation of parameter to solve the corresponding inhomogeneous equation:

$$x^2y''(x) + 7xy'(x) + 9y(x) = x$$

Problem 5. (24 points) For the differential equation $x^2y'' + xy' - 2xy = 0$

- (1) Find and classify (as regular or irregular) the singular point;
- (2) For regular singular point, solve the indicial equation;
- (3) suppose $y(x) = \sum_{n=0}^{\infty} a_n x^n$ is series solution of the equation, find the recurrence relation of the coefficients.

Problem 6. (18 points) For what values of nonnegative α, that is, $\alpha \geq 0$, are there infinitely many solutions to the boundary value problem?

$$y'' + 4\alpha y = 0, \quad y(0) = y(1) = 0$$