MATH 240 SPRING 2015

EXAM 3

Problem 1. (4 pts)

Your name:

Recitation instructor name:

Recitation time:

Problem	1	2	3	4	5	6	7	Total
Grade								

Problem 2. (16 pts) Solve the following initial value problem, using Laplace transform:

$$\ddot{x} + 4\dot{x} + 4x = \delta(t-2), \quad x(0) = \dot{x}(0) = 0.$$

Problem 3. (16 pts) Solve the following system:

$$\begin{cases} \dot{x} = x - y, & x(0) = 1, \\ \dot{y} = 5x - y, & y(0) = 1. \end{cases}$$

Problem 4. (16 pts) Solve as a convolution integral (do not evaluate the integral)

$$\ddot{x} - 3\dot{x} + 2x = e^{\sin t}, \quad x(0) = \dot{x}(0) = 0.$$

Problem 5. (16 pts) Find and classify (as stable, unstable or saddle) the equilibria of the system

$$\begin{cases} \dot{x} = x^2 - 2xy - 3y^2 \\ \dot{y} = x - y - 1 \end{cases}$$

Problem 6. (16 pts) It is known that the general solution of the homogeneous Euler equation

$$x^2y'' - 3xy' + 3y = 0$$

is $y = Ax + Bx^3$, where A and B are arbitrary constants.

Use variation of parameters to find the general solution of the inhomogeneous equation

$$x^2y'' - 3xy' + 3y = x.$$

Problem 7. (16 pts) Let

$$y(x) = \sum_{n=0}^{\infty} c_n x^n$$

be the solution of the initial value problem

$$(x^2+2)y'' + 6xy' + 6y = 0, \quad y(0) = 1, \quad y'(0) = 2.$$

Determine the recurrence relation for the coefficients c_n. Find the coefficients c_0, c_1, c_2, c_3 and a lower bound for the radius of convergence of the power series $y(x)$.

Laplace transform

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1	$\frac{1}{s}$
t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
$\sin(at)$	$\frac{a}{s^2+a^2}$
$\cos(at)$	$\frac{s}{s^2+a^2}$
$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$
$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
$\delta(t-c)$	e^{-cs}
$u(t-c)f(t-c)$	$e^{-cs}F(s)$
$f'(t)$	$sF(s)-f(0)$
$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$\int_0^t f(t-\tau)g(\tau)d\tau$	$\mathcal{L}\{f(t)\}\mathcal{L}\{g(t)\}$