# MATH 240: Exam 1

Name:

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| Problem | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Points | | | | | | | | |

1) Find the solution to the following initial value problem:

$$y' + y = e^x, y(0) = 1$$

.

2) Find the general solution to

$$\frac{dy}{dx} = \frac{y\sin(x) + \frac{y^2}{2}}{\cos(x) - xy}$$

3) Find the solution to

$$y' + \frac{1}{x}y = y^4, x > 0$$

that satisfies the condition $y(1) = \frac{3}{2}$.

4) Find **all** solutions to the differential equation

$$\frac{dy}{dx} = \frac{xy(x+1)}{y^2+1}.$$

You may leave your solution in implicit form.

5) Determine the general solution to the differential equation

$$\frac{dy}{dx} = \frac{\frac{2x}{y} + \frac{y^2}{2x^2}}{\frac{x^2}{y^2} + \frac{y}{x}}.$$

You may leave your solution in implicit form.

6) In a magical kingdom far, far away, a certain number of fairies are born in a hidden garden each year. Unfortunately, the trolls who live nearby hunt the fairies - the more fairies there are, the more they hunt. Thus, the fairy population is modeled by the following differential equation:

$$\frac{dP}{dt} = f - hP,$$

where $f$ and $h$ are positive constants. Find any equilibria of the system, classify them/it as stable, unstable, or semi-stable, and draw the equilibria and a few solution curves (slope field diagram).

7) Show that every separable equation,

$$\frac{dy}{dx} = \frac{f(x)}{g(y)},$$

is an exact equation.

8) Draw the slope field for each equation listed below. Be sure to show enough arrows to reveal the behavior of the solutions.

$$y' = x^2 - 4$$

$$y' = \sin(y)$$

$$y' = x^2 y$$