Name: _____

Recitation: _____

Math 240
Final Exam
December 13, 2017

Problem	Score
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
Total	

Closed book. You may use a calculator and one $8\frac{1}{2} \times 11$ sheet of handwritten notes (both sides). You must show your work to receive full credit. In problems where you are asked to justify your answer, you will be graded on both correctness and clarity. Errors in spelling and grammar may reduce the clarity of your answer.

You may use the Laplace transform table at the end of this exam.

Name: _____

1. (10 pts) For each of the equations below, indicate its order and determine whether it is autonomous (Yes or No) and whether it is linear (Yes or No).

equation	order	autonomous?	linear?
$y''' - xy'' + 6y = \cos(x)$			
$y''' - yy'' + 6y = \cos(y)$			
$y^{(5)} - 4xy^2 = 7$			
$e^{y''} - 2y' = 0$			
$y^{(4)} - 4y'' + 2y = 0$			

Name: _____

2. (10 pts) Convert the second order ODE

$$3x'' + x\cos(x') + 4e^x = 0$$

into a first order system of ODEs.

Name: _____

3. (10 pts) Find *all* values of s so that $y(x) = x^s$ solves the ODE

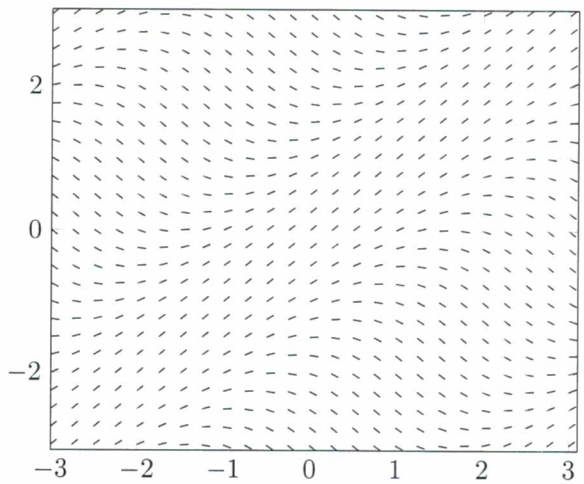
$$2x^2y'' + 7xy' - 3y = 0.$$

Name: _____

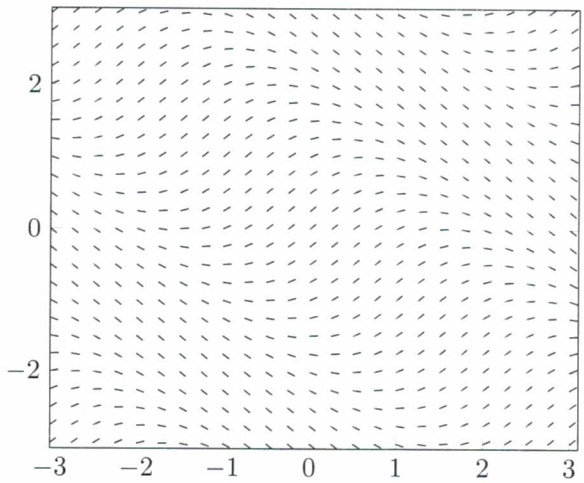
4. (10 pts) Match each of the following 4 differential equations with its corresponding slope field on the following page. Justify your answer.

1. $y' = \cos(x) - \cos(y)$
2. $y' = \cos(xy)$
3. $y' = \cos(x)\cos(y)$
4. $y' = \cos(x) + \cos(y)$

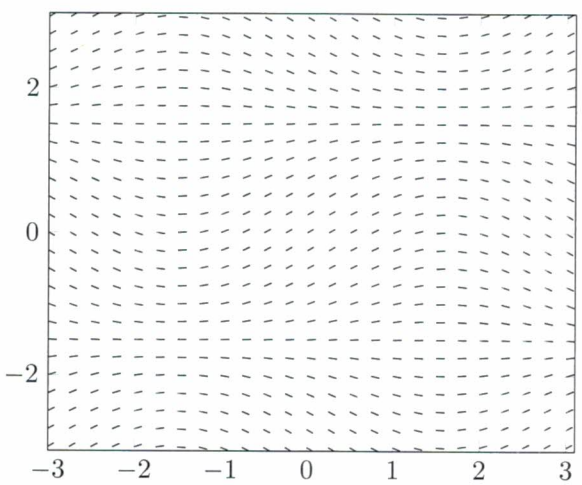
Name: _____


2
0
−2
−3 −2 −1 0 1 2 3

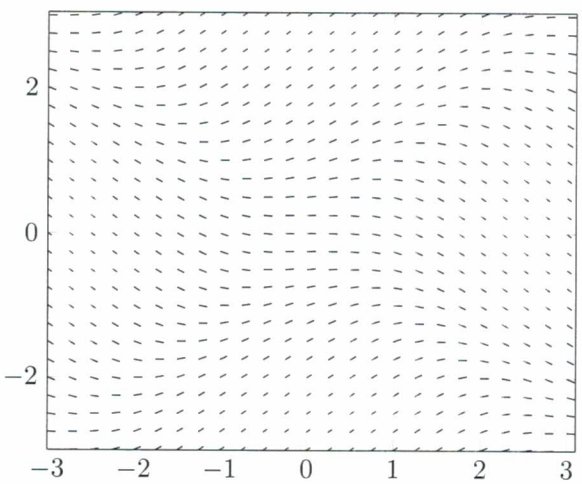
(a)


2
0
−2
−3 −2 −1 0 1 2 3

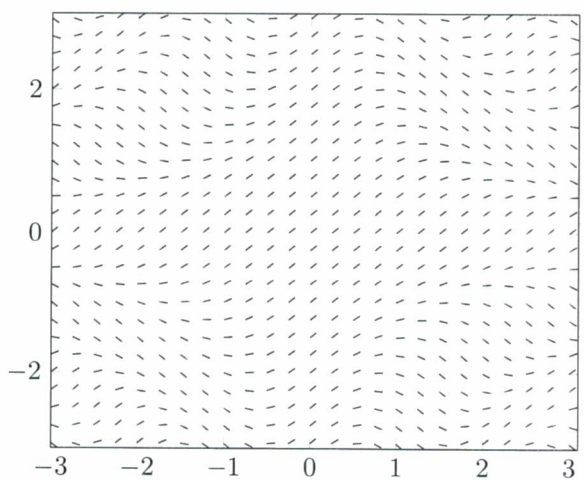
(b)


2
0
−2
−3 −2 −1 0 1 2 3

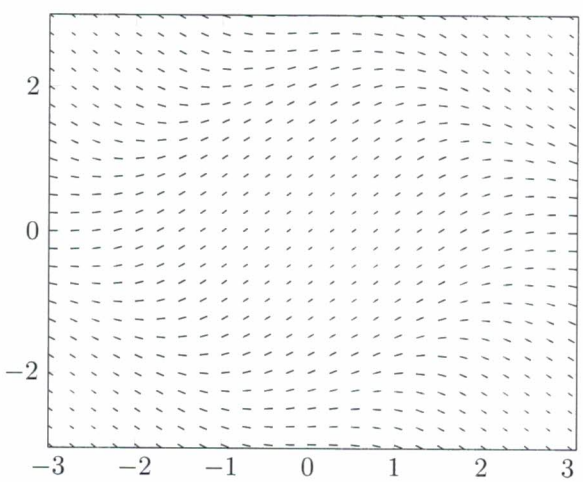
(c)


2
0
−2
−3 −2 −1 0 1 2 3

(d)


2
0
−2
−3 −2 −1 0 1 2 3

(e)


2
0
−2
−3 −2 −1 0 1 2 3

(f)

Name: _____

5. (10 pts) Consider the differential equation

$$y' = 3y(y+1)(y-4)^2.$$

(a) Draw the phase diagram.

(b) Sketch the slope field along with typical solutions.

(c) For which initial values $y(0) = y_0$ is it true that $\lim_{x\to\infty} y(x) = 4$?

Name: _____

6. (10 pts) Find a third order linear constant coefficient differential equation whose general solution is

$$y(x) = c_1 e^{-x} + c_2 e^{-2x} + c_3 x e^{-2x}.$$

Name: _____

7. (10 pts) The general solution of

$$4x'' + 9x = 0$$

is

$$x = c_1 \cos\left(\frac{3}{2}t\right) + c_2 \sin\left(\frac{3}{2}t\right).$$

What is the general solution of

$$4x'' + 9x = 3e^{-\frac{3}{2}t}?$$

Name: _____

8. (10 pts) Match each mass-spring equation on the left with its (x, t) plot on the right. For *each* example, explain why the graph is consistent with the equation. ("Process of elimination" is not a valid explanation.)

1) ____ $3x'' + 5x' + 2x = 0$

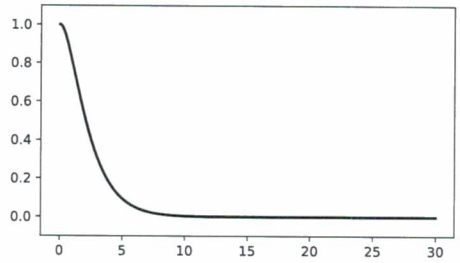
2) ____ $3x'' + 12x = \cos(2t)$

3) ____ $2x'' + 3x' + 5x = 0$

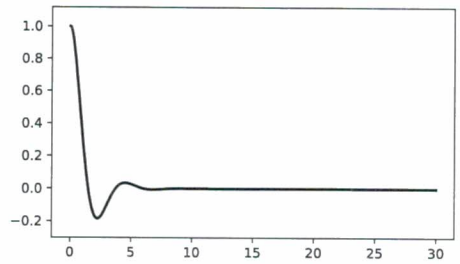
4) ____ $5x'' + 19x = 0$

5) ____ $3x'' + 12x = \cos(3t)$

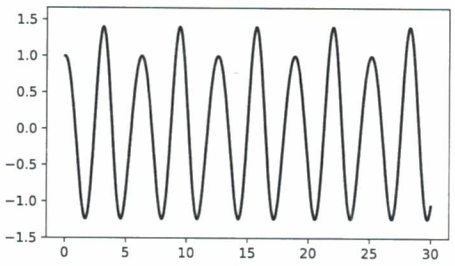
a)



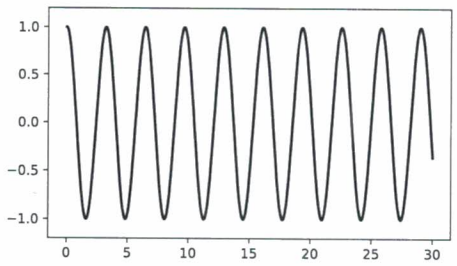
b)



c)



d)



e)