Algebra QE I Syllabus

Designed for students who have had Math 730 and 731.

Groups

Basics Cyclic groups, symmetric groups, matrix groups, Cayley's theorem, dihedral groups, direct products, normal subgroups, homomorphisms, kernels, cosets & Lagrange's theorem, quotient groups, isomorphism theorems, simple groups.

Group actions The class equation, Sylow theorems, groups of small order.

Abelian groups The fundamental theorem of finitely-generated abelian groups.

Rings

Basics Ideals, homomorphisms, quotient rings, matrix rings, characteristics, polynomial rings, quaternions.

Integral Domains Principal ideal domains, Euclidean domains, unique factorization domains.

Linear algebra

Vector spaces Subspaces, linear independence, bases, dimension, linear transformations, dual spaces, rank-nullity theorem, invariant subspaces.

Matrices Determinants, cofactors & inverses, eigenvalues & eigenvectors, diagonalization, how to compute rational and Jordan canonical forms.

Modules

Basics Submodules, homomorphisms, quotient modules, direct sums, free modules.

Modules over PIDs The fundamental theorem of finitely generated modules over PIDs.

Field theory

Basics Algebraic and transcendental extensions, degree of a finite extension, finite fields, cyclotomic extensions, simple extensions.

Extensions Splitting fields, separable extensions, Galois extensions.

Automorphisms Fixed fields, Galois groups, fundamental theorem of Galois theory.