

**1. (10 pts)**

Consider two permutations in S_{100} : let $\sigma = (1\ 2)$ be the transposition exchanging 1 and 2 and let $\tau = (2\ 3\ \dots\ 100)$ be the cycle that sends 2 to 3, 3 to 4, etc. and 100 to 2.

- a) Where does τ^{20} send 18?
- b) Find all permutations that you get by conjugating σ by powers of τ .



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**2. (10 pts)**

- a) Let I be the ideal in $\mathbb{R}[x]$ generated by $x^3 + 2x^2 + x$ and $x^3 + 3x^2 + 3x + 1$. Can this ideal be generated by one element?
- b) Is I a prime ideal?



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**3. (10 pts)**

Let I and J be ideals in a PID R . Suppose $M = R/I$ and $N = R/J$. Show that if $\text{Hom}(M, N) = 0$ then the ideal

$$I + J = \{a + b : a \in I, b \in J\}$$

coincides with the ring R .



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4. (10 pts) Let U , V , and W be complex vector spaces. Let $A : U \rightarrow V$ and $B : V \rightarrow W$ be linear transformations. Given that the dimension of the kernel of A is 2, the dimension of the kernel of BA is 3, and the dimension of V is 4, list all the possibilities for the rank of BA .



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5. (10 pts)

Show that if two complex matrices are similar, then they have the same characteristic polynomial.



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**6. (10 pts)**

Let $F \subset E$ be a Galois extension of fields with the Galois group D_8 (the dihedral group with 8 elements). Find the number of distinct fields K such that $F \subset K \subset E$ and K is a Galois extension of F of index 4. Justify your answer.



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