Analysis QE I Syllabus

Designed for students who have had Math 721 along with either Math 722 or a 700-level complex analysis course.

Metric space topology countability, equivalent definitions of compactness, perfect sets, connected sets.

Sequences and Series Convergence of sequences in metric space, Cauchy sequences, $\liminf$ and $\limsup$, tests for convergence of series, rearrangements of series.

Continuity ϵ-δ definition, open set definition, relationships between continuity and compactness (resp. connectedness), types of discontinuities, monotonicity.

Differentiation mean value theorems, L'Hôpital's Rule, Taylor's Theorem

Integration Riemann integration, Riemann-Stieltjes integration, Fundamental Theorem of Calculus.

Sequences and Series of Functions Preservation of properties (integrability, differentiability, continuity) under pointwise convergence and under uniform convergence, equicontinuity, Stone-Weierstrass Theorem, power series, Taylor Series, Fourier Series.

Functions of several variables Differentiation, fixed-point theorem, Inverse Function Theorem, Implicit Function Theorem.

Analytic functions Power series, Möbius transformations, Cauchy-Riemann equations.

Complex integration Cauchy's Integral Formula, Residue Theorem, countour integration, Laurent Series.

Zeros Liouville's Theorem, Maximum Modulus Theorem, Open Mapping Theorem.