Analysis QE I Syllabus – March 2019

Covers material from Math 721 and Math 723.

Basics Metric space topology, Compactness, Connectedness, Completeness, Convergence, Series of real numbers.

Analysis in $\mathbb{R}^1$ Differentiation, Mean value theorems, L'Hôpital's Rule, Taylor's Theorem.

Lebesgue Measure Lebesgue measure, Integration, Relationship between Lebesgue and Riemann integration, L^1 functions, Dominated Convergence Theorem, Fatou's Lemma.

Sequences of Functions Preservation of properties (integrability, differentiability, continuity) under pointwise convergence and under uniform convergence, Equicontinuity, Arzelà-Ascoli Theorem

Functions of several variables Continuity, Relationships between continuity and compactness (resp. connectedness), Differentiation, Inverse Function Theorem, Implicit Function Theorem.

Analytic functions Power series, Cauchy-Riemann equations.

Complex integration Cauchy's Integral Formula, Contour integration, Laurent Series.

Other topics Liouville's Theorem, Zeros of holomorphic functions, Maximum Modulus Theorem, Classification of isolated singular points.