Analysis QE I June 2016

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1. (10 pts)

Suppose that $(a_n)_{n=1}^{\infty}$ is a convergent sequence of real numbers. Let $b \in \mathbb{R}$ be such that

$$\forall n \geq 1,\ a_n \neq b \qquad \text{and} \qquad \lim_{n\to\infty} a_n \neq b.$$

Show that there must be a $d > 0$ such that $\forall n \geq 1$, $|a_n - b| > d$.

2. (10 pts)

- Let $(x_n)_{n=1}^{\infty}$ be a sequence of real numbers such that $\sum_{n=1}^{\infty} x_n$ converges, but $\sum_{n=1}^{\infty} x_n^2$ diverges. Prove that $\sum_{n=1}^{\infty} x_n$ must converge conditionally.
- Let (x_j) and (y_j) be sequences of real numbers such that $\sum_{j=1}^{\infty} x_j$ and $\sum_{j=1}^{\infty} y_j$ are both convergent. Prove that the series $\sum_{j=1}^{\infty} \sqrt{|x_j y_j|}$ is also absolutely convergent. Hint: a possible solution uses the Limit Comparison Test.