1). Let $a < b$ be two real numbers, and let $f(x)$ be a real-valued function, continuously differentiable on the interval $[a, b]$. Prove that $f(x)$ is uniformly differentiable on $[a, b]$ – that is, for every real $\epsilon > 0$ there is $\delta > 0$, such that the inequality

$$\left| \frac{f(x) - f(y)}{x - y} - f'(x) \right| < \epsilon$$

holds whenever $a \leq x \leq b$, $a \leq y \leq b$, $0 < |x - y| < \delta$.

2). Let (X, Σ, μ) be a measure space, and let $f : X \longrightarrow [-\infty, +\infty]$ be a measurable function. For real $t > 0$ consider set

$$S_t = \{x \in X : |f(x)| \geq t\}.$$

Prove Chebyshev's inequality:

$$\mu(S_t) \leq \frac{1}{t} \int_X |f| d\mu.$$

3). Prove that function $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$, given by

$$f(x, y) = \begin{cases} \dfrac{x^3 - y^3}{x^2 + y^2}, & \text{if } (x, y) \neq (0, 0), \\ 0, & \text{if } x = y = 0, \end{cases}$$

is continuous, but not differentiable, at $(0, 0)$.

4). Let $f(z)$ and $g(z)$ be two entire functions, such that

$$|f(z)| \leq |g(z)|, \quad z \in \mathbb{C}.$$

Prove that there is a constant $\lambda \in \mathbb{C}$, such that

$$f(z) = \lambda g(z), \quad z \in \mathbb{C}.$$

5). Calculate the coefficient c_{-1} in the Laurent expansion

$$\frac{1}{e^z - 1} = \sum_{n=-\infty}^{+\infty} c_n z^n, \quad 2\pi < |z| < 4\pi.$$

6). Let $f(z)$ be a linear-fractional transformation, such that $f(1) = 0$, $f(i) = \infty$, and $f(-i) = 1$. Describe the image $f(D)$ of the domain

$$D = \{z \in \mathbb{C} : |z| < 1, |z - 1 - i| < 1\}.$$