Applied Mathematics Qualifying Exam Syllabus

- **Vector space concepts:** dimension (finite/infinite), bases, complete orthonormal sets; properties of norms and inner products; definition of Banach space, Hilbert space
- **Direct methods for linear systems:** solvability criteria; condition number and its implications; LU, Cholesky, basic ideas of computational complexity
- **Least squares problems:** solving with normal equations, QR, SVD; low-rank approximation with SVD
- **Eigenvalue problems:** defective eigenvalues and stability implications; spectral theorem for symmetric or Hermetian matrices
- **Function spaces:** vector spaces, normed spaces, convergence, completeness; Banach spaces; Hilbert spaces; L^p spaces (no measure theory)
- **Hilbert space techniques:** complete orthonormal sets and generalized Fourier series; least squares, orthogonal projection
- **Linear operators:** boundedness; bounded linear functionals, Riesz representation theorem; adjoint operators; Fredholm Alternative
- **Compact operators:** spectral theorem for compact self-adjoint operators; resolvent operators and connection to spectrum
- **Differential operators and distributions:** test functions and distributions; distributional derivatives; Green's functions for differential equations

References

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- Trefethen, Lloyd N., and David Bau III. *Numerical linear algebra.* SIAM, 1997.
- Golub, Gene H., and Charles F. Van Loan. *Matrix computations.* JHU Press, 2012.
- Keener, James P. *Principles of Applied Mathematics.* Addison-Wesley, 1995.
- Kreyszig, Erwin. *Introductory functional analysis with applications.* Wiley, 1989.