



1. a) Define the subspace topology.
- b) Let  $f : X \rightarrow Y$  be a continuous map, let  $X \times Y$  be the product space (with the product topology) and let

$$\Gamma(f) = \{(x, f(x)) \mid x \in X\} \subset X \times Y,$$

equipped with the subspace topology. Show that  $\Gamma(f)$  is homeomorphic to  $X$ .



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Topology QE I August 2016

#1 4 of 18



2. Let  $X$  be a topological space, and let  $\sim$  be a relation on  $X$ .
- a) State what it means for  $\sim$  to be an equivalence relation.
  - b) Define  $X/\sim$  and the quotient topology on it.
  - c) True or false (with proof or counter-example): If  $X$  is compact,  $X/\sim$  is compact.
  - d) True or false (with proof or counter-example): If  $X$  is normal,  $X/\sim$  is normal.



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Topology QE I August 2016

#1            6 of 18



3. Recall that a map  $f : X \rightarrow Y$  is *open* if  $f(U)$  is open whenever  $U \subseteq X$  is open. Recall that a *closed map* is one for which  $f(A)$  is closed whenever  $A \subseteq X$  is closed. Let

$$S^2 := \{v \in \mathbb{R}^3 \mid |v| = 1\}.$$

Give  $S^2$  the subspace topology of the metric topology on  $\mathbb{R}^3$ . Let  $g : S^2 \rightarrow S^2$  be continuous and open.

- a) Prove that  $g$  is a closed map.
- b) Prove that  $g$  is surjective.



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Topology QE I August 2016

#1            8 of 18



4. a) Define what it means for a map to be nullhomotopic.
- b) Prove that if  $|\pi_1(X)| < \infty$ , then every map  $f : X \rightarrow T^2$  is nullhomotopic (where  $T^2 = S^1 \times S^1$  is the 2-torus).



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Topology QE I August 2016

#1            10 of 18





5. Let

$$SK := \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 \leq 1, |z| \leq 1\} / \sim \quad \text{and}$$

$$K := \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 = 1, |z| \leq 1\} / \sim \quad \text{where}$$

$(x, y, -1) \sim (x, -y, 1)$ . Let  $[(1, 0, 1)]$  be the base point for each space. Let  $\iota : K \rightarrow SK$  be the inclusion.

- a) Compute  $\pi_1(K)$ ,  $\pi_1(SK)$ , and the induced map  $\iota_* : \pi_1(K) \rightarrow \pi_1(SK)$ .
- b) Construct a non-cyclic abelian group  $H$  and a surjective homomorphism  $f : \pi_1(K) \rightarrow H$ .
- c) Prove that there is no continuous map  $r : SK \rightarrow K$  so that  $\iota \circ r = \text{id}_K$ .



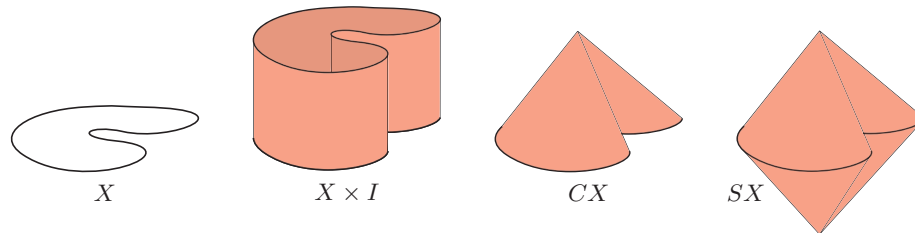
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Topology QE I August 2016

#1            12 of 18



- 6. Definition:** Given a space  $X$ , the *cone*  $CX$  is the quotient space obtained by collapsing one end of the cylinder  $X \times I$ . That is  $X \times I / \sim$  where  $(x, 1) \sim (y, 1)$  for all  $x, y \in X$ . Furthermore, the *suspension*  $SX$  is the quotient space obtained by collapsing both ends of the cylinder  $X \times I$  separately - or equivalently by identifying two copies of  $CX$  along  $X \times \{0\}$ .



**Problem:** Calculate the cohomology of  $SX$  in terms of the cohomology of  $X$  (Hint: It is helpful to note that  $CX$  is contractible).



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Topology QE I August 2016

#1            14 of 18

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Topology QE I August 2016

#1 15 of 18





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Topology QE I August 2016

#1            16 of 18

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Topology QE I August 2016

#1 17 of 18





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Topology QE I August 2016

#1            18 of 18