1. (10 pts)

Recall that the long line L is the set $(\Omega+1)\times[0,1)\setminus\{(0,0)\}$ where Ω is the first uncountable ordinal (so $\Omega+1$ is the set of all ordinals from $0=\emptyset$ up to and including Ω, ordered by $\in$), with the order topology induced by the lexicographic order $(\alpha,x)\leq(\beta,y)$ whenever $\alpha<\beta$ or both $\alpha=\beta$ and $x\leq y$.

a) Prove in detail that L is connected.

b) Which of the following topological properties does L exhibit – Hausdorffness, separability, second countability, compactness, path connectedness? For each explain *briefly* why it does or does not exhibit the property.

For part (b) your explanations need not be complete proofs, just an indication that you understand the key idea that could be exploited to give a proof, or citing a theorem that implies your claim.

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2. (10 pts)

a) State the Meyer-Vietoris theorem for deRham cohomology.

b) Use the Meyer-Vietoris theorem, homotopy invariance, the well-known deRham cohomology groups of spheres and contractible spaces and a suitable open cover to compute the deRham cohomology of the space obtained by removing a closed disk from the torus $S^1 \times S^1$.

3. (10 pts) Let ω denote the form $xdz \wedge dy + y^3 dx \wedge dy + z^2 dy \wedge dz$ on $\mathbb{R}^3$. Calculate the integral of $\omega|_{S^2}$ over the standard unit sphere $S^2 \subset \mathbb{R}^3$.

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4. (10 pts)

Find the fundamental group of the space obtained by attaching two disks (thought of as the unit disk in the complex plane) to the torus $S^1 \times S^1$ (thinking of S^1 as the unit circle in the complex plane) by their boundaries, the first by the map $e^{i\theta} \mapsto (e^{im\theta}, 1)$, the second by the map $e^{i\theta} \mapsto (1, e^{in\theta})$ for m, n non-zero integers. A presentation by generators and relations will be acceptable, but it should be possible to identify the group as a reasonably familiar finite group for every pair m, n.

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5. (10 pts)

a) Define what it means for a space to be compact.

b) Define what it means for a space to be sequentially compact.

c) Prove that if a X is compact then X is sequentially compact.

6. (10 pts)

a) Define what it means for a smooth map between manifolds $f : X \to Y$ to be an immersion.

b) What additional conditions must an immersion statify to be an embedding?

c) Using the inverse function theorem as a starting point, prove that an immersion is locally injective (that is, for any $x \in X$ there is a neighborhood of x on which the immersion f is one-to-one).

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