Topology QE I Exam
June 2019

Name ....

KSU Email ....

Instructions:

Do not write your name or any other identifying information on any page except this cover page.

Use the space below the statement of a problem as well as the back of the page and the next page for the solution. If more space is needed, use the blank pages at the end.

All pages must be submitted. If there is work you want ignored, cross it out (or otherwise indicate its status) or tape a clean sheet over it to create more space to be used, being careful not to cover the code at the top.

You have three hours to work on these problems. Attempt all problems. Four complete solutions will earn a pass. Credit for completed parts of separate problems may combine to result in a pass.

No references are to be used during the exam.

1. (10 pts)

Consider the set $\mathbb{R} \setminus \mathbb{Q}$ of irrational numbers endowed with the cofinite topology (only finite sets and the entire set are closed).

(a) Prove that it is compact.

(b) Prove that it is path-connected.

(c) Is it T_0? T_1? Hausdorff? regular? normal? connected? metrizable?

2. (10 pts)

(a) Give an example of a connected 2-sheeted covering of the wedge of two circles and a 2-sphere $S^1_a \vee S^1_b \vee S^2$.

(b) Is your example a regular covering?

3. (10 pts)

(a) Compute $\pi_1(S^1 \times \mathbb{R}P^2)$.

(b) Let X be the space obtained from two copies of $S^1 \times \mathbb{R}P^2$ by identifying them along their circles $S^1 \times \{p\}$, where p is the base point of $\mathbb{R}P^2$. Compute $\pi_1(X)$.

4. (10 pts)

Let X be the real line $\mathbb{R}$ with the standard smooth structure. Let Y denote the same real line with the smooth structure given by the maximal atlas of the coordinate chart $\psi\colon \mathbb{R} \to \mathbb{R}$, $\psi(x) = x^{\frac{1}{5}}$. Show that the two smooth structures on $\mathbb{R}$ are distinct, but that X is diffeomorphic to Y.

5. (10 pts)

Let $f_1, \ldots, f_n$ be smooth functions on a neighborhood U of a point p. Show that there is a neighborhood $W \subset U$ of p on which $f_1, ..., f_n$ are coordinate functions if and only if $df_1 \wedge \ldots \wedge df_n|_p \neq 0$.

6. (10 pts)

Calculate the de Rham cohomology groups of an orientable surface of genus 2. (You may use the fact that the cohomology groups of a punctured torus are R in degree 0, $\mathbb{R}^2$ in degree 1, and 0 otherwise.)