Name: Key

Recitation Instructor and Time: _____

Studio College Algebra – Exam 3
November 12, 2013

Please show all your work for full credit. Every problem is worth 5 points.

1. Rewrite the formula $y = 211x$ by taking the natural logarithm of both sides, and expand wherever possible using properties of logarithms (no decimal approximations).

$\ln y = \ln(211x)$

$\boxed{\ln y = \ln 211 + \ln x}$

2. Solve for S in the following equation: $\ln\left(\frac{S}{3500}\right) = 2$

(Leave answer in exact form, no decimal approximations please).

$e^2 = \frac{S}{3500}$

$\boxed{3500e^2 = S}$

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3. If $\log(a) = 4.2$ and $\log(b) = 3.8$, what is $\log\left(\sqrt[3]{ab}\right)$?

$\log\left(\sqrt[3]{ab}\right) = \log(ab)^{1/3}$

$= \frac{1}{3}[\log(ab)]$

$= \frac{1}{3}[\log(a) + \log(b)]$

$= \frac{1}{3}[4.2 + 3.8] = \frac{1}{3}[8] = \boxed{\frac{8}{3}}$ or $2.\overline{66}$

4. Solve $4 + 5^x = 9$

$5^x = 5$ → By inspection, $\boxed{x = 1}$

$\ln 5^x = \ln 5$

$x \ln 5 = \ln 5$

5. Solve $9\ln(x+5) = 27$.

$\ln(x+5) = 3$

$e^3 = x+5$

$\boxed{e^3 - 5 = x}$ or $x \approx 15.086$.

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6. What is the future value in 10 years of an initial investment of $800 at an annual interest rate of 6%, compounded monthly?

$$\text{Future Value} = (\text{Present Value})\left(1 + \frac{r}{n}\right)^{nt}$$

r = interest rate.
t = # of yrs.
n = # of compoundings per yr.

$$\text{Future Value} = 800\left(1 + \frac{.06}{12}\right)^{12(10)}$$

$$= 800(1.005)^{120}$$

$$= \boxed{\$1455.52}$$

7. A certain type of bacteria grows according to the function $P(x) = 3500(2^x)$, where x is the number of hours that have passed by. How many bacteria will there be after 3 hours?

$$P(3) = 3500(2^3)$$

$$= 3500(8)$$

$$= \boxed{28000 \text{ bacteria}}$$

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8. The number of widgets (in thousands) demanded each year is given by the formula $D(x) = 5 + 10\log(x+3)$, where x represents the number of years after 1980, and $x > 0$. In what year were 25,000 widgets demanded?

$25 = 5 + 10\log(x+3)$

$20 = 10\log(x+3)$

$2 = \log(x+3)$

$10^2 = x + 3$

$100 = x + 3$

$97 = x.$

$1980 + 97 = 2077$

In year 2077

9. Find a possible 3rd degree polynomial with single roots at $x = 4, x = -1$, and $x = 3$. Do not multiply your answer out.

$(x-4)(x+1)(x-3)$

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10. Given that -5 is a solution, find all solutions, both real and complex, of the following equation: $x^3+5x^2+3x+15=0$.

$$\begin{array}{r|rrrr} -5 & 1 & 5 & 3 & 15 \\ & \downarrow & -5 & 0 & -15 \\ \hline & 1 & 0 & 3 & 0 \end{array}$$

$x^2+3=0$

$x^2=-3$

$x=\pm\sqrt{3}\,i$

$x=-5,\ \pm\sqrt{3}\,i$

11. Is $x-3$ a factor of $x^3-5x^2+10x-7$? How do you know?

$$\begin{array}{r|rrrr} 3 & 1 & -5 & 10 & -7 \\ & \downarrow & 3 & -6 & 12 \\ \hline & 1 & -2 & 4 & 5 \end{array}$$

The remainder, after dividing $x^3-5x^2+10x-7$ by $x-3$ is not 0. So, $x-3$ is not a factor.

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12. Given that $x = -5$ and $x = 3$ are roots of the following polynomial, find all other roots, real and complex, of the polynomial: $f(x) = x^4 + 4x^3 - 7x^2 - 22x - 60$

$$\begin{array}{r|rrrrr} -5 & 1 & 4 & -7 & -22 & -60 \\ & \downarrow & -5 & 5 & 10 & 60 \\ \hline & 1 & -1 & -2 & -12 & 0 \\ 3 & \downarrow & 3 & 6 & 12 & \\ \hline & 1 & 2 & 4 & 0 & \end{array}$$

$$x^2 + 2x + 4 = 0$$

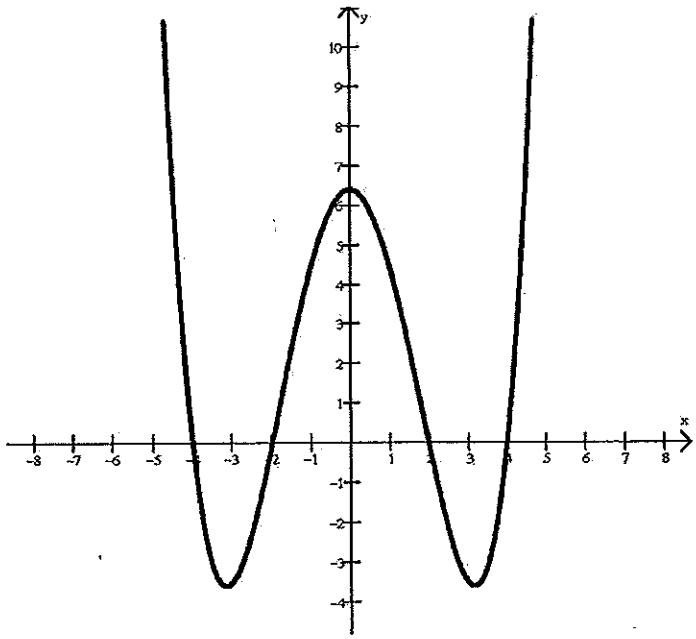
$$x = \frac{-2 \pm \sqrt{4 - 4(1)(4)}}{2}$$

$$x = \frac{-2 \pm \sqrt{-12}}{2}$$

$$x = \frac{-2 \pm 2\sqrt{3}\,i}{2}$$

$$x = -1 \pm \sqrt{3}\,i$$

13. Given the graph on the right, decide whether the following statements are **True or False**. You may assume nothing interesting happens outside the window shown.



a) This polynomial has a negative leading coefficient.

False.

b) The polynomial has a positive constant term.

True.

c) The polynomial does not have any repeated roots.

True.

d) The polynomial has even degree.

True

e) As x tends to both positive and negative infinity, the polynomial tends towards negative infinity.

False.

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14. What is the domain of the function $f(x) = 4 + 3\ln(-7x + 9)$?

$-7x + 9 > 0$

$-7x > -9$

$x < \frac{9}{7}$

15. What is the horizontal asymptote of the function $f(x) = e^x - 6$? Briefly explain how you arrived at your answer. Your explanation should use the methods of this class and not rely solely on a graphing calculator.

$y = -6$

Reason: $y = e^x$ has a horizontal asymptote at $y = 0$. The -6 in $f(x) = e^x - 6$ shifts the graph of $y = e^x$ down 6 units, so the horizontal asymptote also shifts down 6 units.

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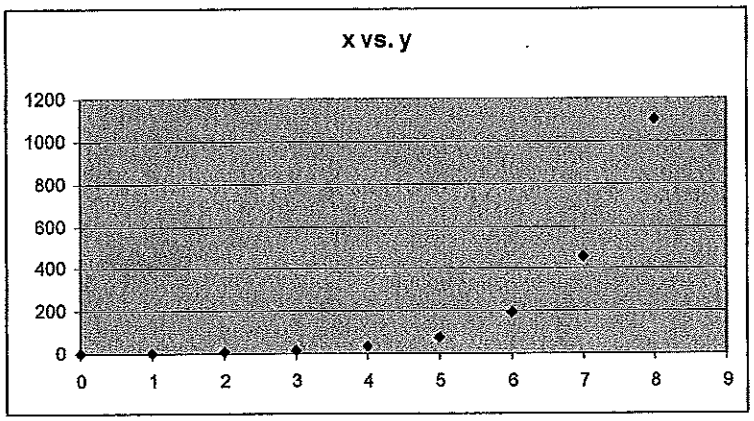
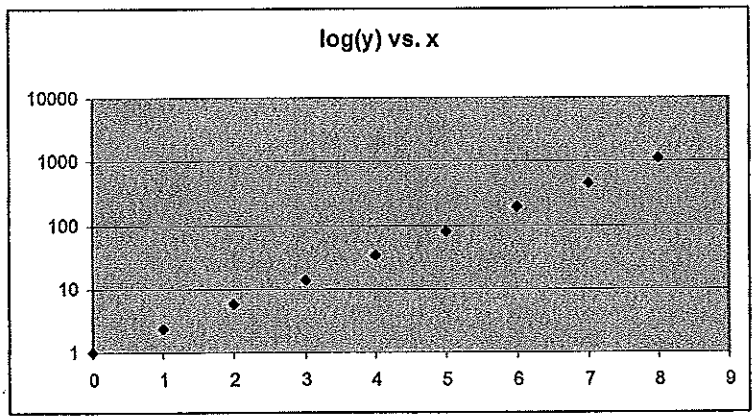
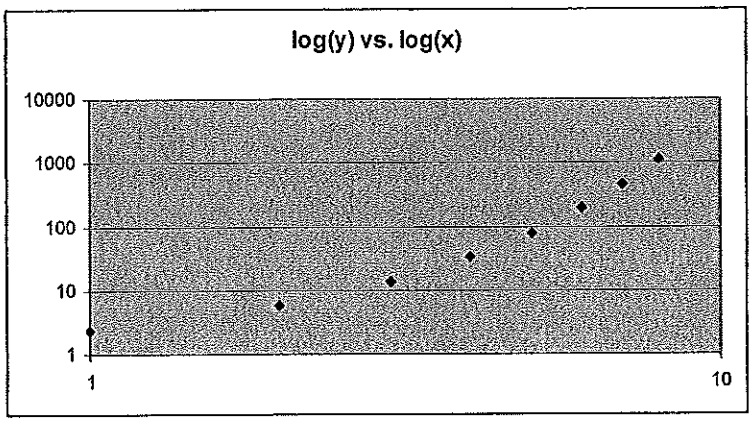
16. Some students have a data set, for which they create standard, log-log, and semi-log plots. (The plots are given below). Would a power or exponential model would be an appropriate fit for the data set? How do you know?







Exponential Model, since the semi-log plot displays a linear pattern.