Name: (Key)
Recitation Instructor:
Recitation Day and Time:

Studio College Algebra – Exam 2 – October 13, 2015

Directions: You will find 16 problems listed below. Each problem is worth 5 points. No notes/books/friends are allowed. Graphing calculator models above the level of a TI-84 plus are not allowed (in particular, calculators with a built in CAS and/or QWERTY keyboard are not allowed). You have one hour to complete this exam. SHOW ALL WORK! You must justify all your answers.

1. Solve $(t-6)^2 = 19$.

$$t - 6 = \pm\sqrt{19}$$

$$\boxed{t = 6 \pm \sqrt{19}}$$

2. Write $x^2 - 10x + 4$ in the form $a(x-h)^2 + k$.

$$x^2 - 10x + 25 - 25 + 4$$

$$= \boxed{(x-5)^2 - 21}$$

3. A parabola has vertex at $(-3, 7)$ and passes through the point $(1, 14)$. What is the equation of the parabola? Write your answer in the form $y = a(x - h)^2 + k$.

$$14 = a(1 - (-3))^2 + 7$$

$$7 = 16a$$

$$a = \frac{7}{16}$$

$$\boxed{y = \frac{7}{16}(x + 3)^2 + 7}$$

4. Find the zeros of the function $s(t) = t^2 + 9t + 4$.

$$0 = t^2 + 9t + 4$$

$$t = \frac{-9 \pm \sqrt{81 - 4(1)(4)}}{2}$$

$$t = \frac{-9 \pm \sqrt{81 - 16}}{2}$$

$$t = \frac{-9 \pm \sqrt{65}}{2}$$

5. Given $h(x) = 4x - 7$ and $k(x) = 3x^2 - 8x$, find $k(x) - h(x)$. (Do not try to factor the final answer.)

$$K(x) - h(x) = 3x^2 - 8x - (4x - 7)$$
$$= 3x^2 - 12x + 7$$

6. Given $r(x) = 5x + 7$ and $m(x) = x^3 + 4x$, find $r(x)m(x)$. (Do not try to factor the final answer.)

$$r(x) m(x) = (5x + 7)(x^3 + 4x)$$
$$= 5x^4 + 7x^3 + 20x^2 + 28x.$$

7. Consider the functions, $f(x) = 4$ and $g(x) = x + 3$:

(a) Using the functions above, find $f(2) + g(2)$.

$f(2) = 4$
$g(2) = 5$

$f(2) + g(2) = \boxed{9}$

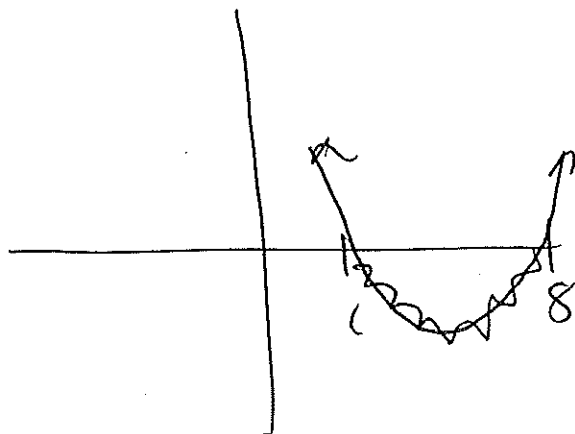(b) Using the functions above, find $f(g(f(3)))$.

~~$f(3) = 4$~~
~~$g(4) = 7$~~

$f(g(f(3)))$

$= f(g(4))$

$= f(7) = \boxed{4}$

8. Solve the quadratic inequality $x^2 - 9x + 8 < 0$.

$(x-8)(x-1) < 0$

$\boxed{1 < x < 8}$

Solution

4

9. Given $f(x) = \dfrac{6x + 7}{5}$, find $f^{-1}(x)$.

$$y = \frac{6x+7}{5}$$

$$5y = 6x + 7$$

$$\frac{5y - 7}{6} = \cancel{x}$$

$$\boxed{f^{-1}(x) = \frac{5x - 7}{6}}$$

10. Solve and check: $x = \sqrt{12 + x}$

$$x^2 = 12 + x$$

$$x^2 - x - 12 = 0$$

$$(x-4)(x+3) = 0$$

$$x = 4 \text{ or } x = -3.$$

$$x = -3: \quad -3 \neq \sqrt{9}$$

$$x = 4: \quad 4 = \sqrt{16}.$$

$$\boxed{\text{Only } x = 4 \text{ works}}$$

11. The height above the water's surface, in feet, of a sea creature's jump, is given by the function $h(t) = -.25(t - 3.5)^2 + 14$, where t is in seconds. When does the creature reach it's highest point, and, how high is the creature at this time?

Since this quadratic opens ↙ $a < 0$, down, the graph (function) achieves a max @ the vertex.

$h(t)$ is in vertex form !!

max ht of 14 ft @ $t = 3.5$ seconds

12. A circular oil spill has a radius (in millimeters) that grows according to the function $R(t) = 3t + 2$, where t is measured in seconds. If the area of a circle of radius r is given by $A(r) = \pi r^2$, find $A(R(t))$. Present your answer in the form $A(R(t)) = at^2 + bt + c$.

$$A(R(t)) = A(3t+2)$$
$$= \pi(3t+2)^2$$
$$= \pi[9t^2 + 12t + 4]$$
$$= 9\pi t^2 + 12\pi t + 4\pi$$

13. If $f(x) = |x|$, describe how the graph of $f(x-6) - 8$ looks.

$f(x-6) - 8$ ~~is~~ has the shape of $f(x)$, but shifted right 6 units and down 8 units.

14. Insect resting metabolic rate (RMR) has been found to be scaled positively with body mass (M) according to the equation $RMR = 4.14(M^{0.66})$, where M is measured in mg and RMR is measured in $mm^3 O_2$ per hour. Find the RMR of an insect weighing 2.5 grams.

$$RMR = 4.14(2500)^{.66}$$
$$= \underline{723.84 \ mm^3 O_2 / hr}.$$

15. Answer the following either by filling in the blank or circling the correct answer.

   (a) The horizontal line test determines whether a relation is __1-1 (or. invertible)__ __one-to one__

   (b) $f(x) = x^3$ and $g(x) = \sqrt[3]{x}$ are __inverses__ of each other.

   (c) If you reflect the graph of $f(x) = x^2, x \geq 0$ over the line __$y = x$__, the resulting graph is that of $g(x) = \sqrt{x}$.

   (d) (True) or False: As long as $m \neq 0$, functions of the form $f(x) = mx + b$, where b is some parameter, are one-to-one functions.

   (e) (True) or False: $f(x) = (x - 2)^2, x \geq 2$, is a one-to-one function.

16. Consider the following piecewise function. Write TRUE or FALSE beside each of the statements given below.

$$f(x) = \begin{cases} 1, & x \leq -3 \\ x^5, & -3 < x \leq 2 \\ -x, & x > 2 \end{cases}$$

   (a) $f(2) = -2$. False
   (b) $f(2) = 32$. True
   (c) $f(-3) = 1$. True
   (d) $f(-3) = -1$. False
   (e) $f(-6) = 1$. True .