Name: Key
Recitation Instructor:
Recitation Day and Time:

Studio College Algebra – Exam 3 – November 15, 2015

Directions: You will find 16 problems listed below. Each problem is worth 5 points. No notes/books/friends are allowed. Graphing calculator models above the level of a TI-84 plus are not allowed (in particular, calculators with a built in CAS and/or QWERTY keyboard are not allowed). You have one hour to complete this exam. SHOW ALL WORK!

1. Fill in the following properties of logarithms (you may assume x and y are positive):

   (a) $\log_b(xy) =$ $\log_b(x) + \log_b(y)$

   (b) $\log_b\left(\frac{x}{y}\right) =$ $\log_b(x) - \log_b(y)$

   (c) $\log_b(x^n) =$ $n\log_b(x)$

2. Fill in the blank. Do not use your calculator.

   (a) $\log_b\left(\sqrt{b}\right) =$ $1/2$

   (b) $\log_3\left(\frac{1}{27}\right) =$ -3

   (c) $\ln(e^6) =$ 6

3. If $\log(a) = 2.6$ and $\log(b) = 1.4$, find $\log\left(a\sqrt[3]{b}\right)$.

$$\log(a\sqrt[3]{b}) = \log(a) + \log(\sqrt[3]{b})$$
$$= \log(a) + \frac{1}{3}\log(b)$$
$$= 2.6 + \frac{1.4}{3}$$
$$= \frac{7.8}{3} + \frac{1.4}{3}$$
$$= \frac{9.2}{3} = \frac{92}{30} = \boxed{\frac{46}{15}} \text{ or } 3.0\overline{6}$$

4. What lump sum would need to be invested at an annual interest rate of 3.5%, under continuous compounding, for 7 years, in order to end up with \$8000? Round answer to the nearest cent.

$$FV = PVe^{rt}$$
$$8000 = PVe^{.035(7)}$$
$$\frac{8000}{e^{.035(7)}} = PV$$
$$\boxed{PV = \$6261.64}$$

5. Solve $3^{x-1} = 15$. Leave answer exact, i.e., do not use calculator.

$\ln 3^{x-1} = \ln 15$

$(x-1)\ln 3 = \ln 15$

$x - 1 = \frac{\ln 15}{\ln 3}$

$x = \frac{\ln(15)}{\ln(3)} + 1$

or in general

$x = \frac{\log_b(15)}{\log_b(3)} + 1$

where b is a valid base.

6. Solve $9\ln(x-3) + 4 = 14$. Leave answer exact, i.e., do not use calculator.

$9\ln(x-3) = 10$

$\ln(x-3) = \frac{10}{9}$

$e^{10/9} = x - 3$

$e^{10/9} + 3 = x$

or

$9\ln(x-3) = 10$

$\ln(x-3) = \frac{10}{9}$

$e^{\ln(x-3)} = e^{10/9}$

$x - 3 = e^{10/9}$

$x = e^{10/9} + 3$

7. Given $f(x) = \log_3(x-5)$, find $f^{-1}(x)$.

$y = \log_3(x-5)$

$3^y = x-5$

$3^y + 5 = x$

Answer: $f^{-1}(x) = 3^x + 5$

8. Find the domain of $f(x) = \log(25+4x)$.

$25 + 4x > 0$

$4x > -25$

$x > -25/4$

9. The function $P(t) = 21.109 - 5.686\ln(t+1)$ describes the revenue, in thousands of dollars, for the sale of a product t weeks after an ad campaign for the product ended, where $0 \le t \le 10$. Find $P(7)$, round to the nearest cent, and interpret the meaning of $P(7)$ in a complete sentence.

$P(7) = 21.109 - 5.686 \ln(7+1)$

$= 9.28530$

7 weeks after the ad campaign ended, the revenue was \$9285.30

10. Find all the zeros of $p(x) = 2x^4 + 3x^3 - 33x^2 + 13x + 15$, given that $x = 3$ and $x = 1$ are zeros of $p(x)$.

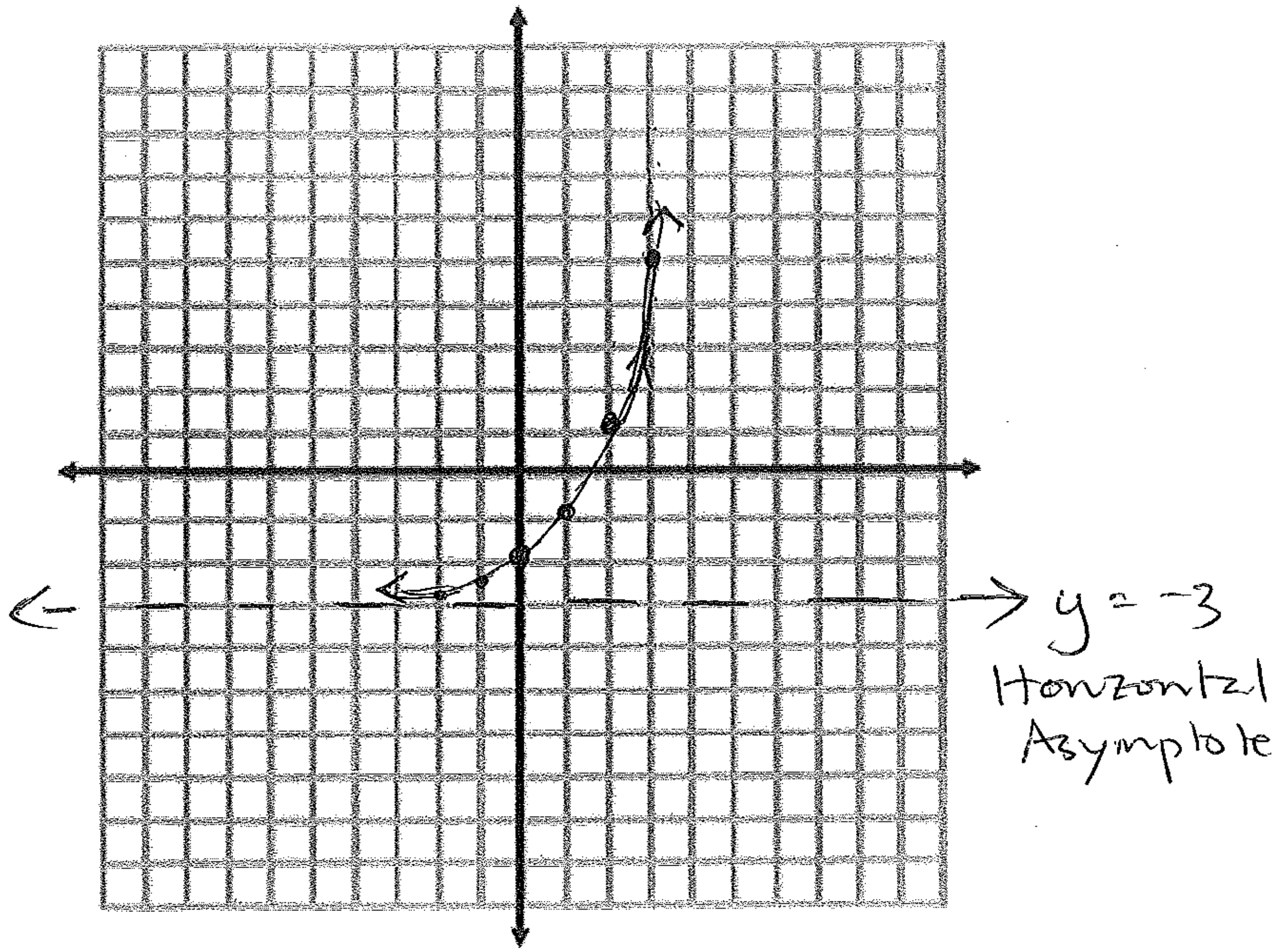
3)	2	3	−33	13	15
	↓	6	27	−18	−15
1)	2	9	−6	−5	0
	↓	2	11	5	
	2	11	5	0	

$2x^2 + 11x + 5 = 0$

$(2x+1)(x+5) = 0$.

$x = -1/2$, $x = -5$, along with $x = 3$ & $x = 1$.

11. Sketch a graph of $f(x) = 2^x - 3$. Include at least 5 plotted points, and any other relevant features.



12. Find an equation for a fourth degree polynomial that passes through the point $(7, -4)$ also having single roots at $x = 1$, $x = 5$ and a double root at $x = -3$. Do not multiply your answer out.

$p(x) = K(x-1)(x-5)(x+3)^2$; plug in $(7,-4)$ & solve for K.

$-4 = K(7-1)(7-5)(7+3)^2$

$-4 = K(6)(2)(100)$

$\frac{-4}{1200} = K$

$K = \frac{-1}{300}$

Answer:

$p(x) = \frac{-1}{300}(x-1)(x-5)(x+3)^2$

13. Solve the polynomial equation $x^3 - 6x^2 - 40x = 0$.

$$x(x^2 - 6x - 40) = 0$$

$$x(x-10)(x+4) = 0$$

$$\boxed{x = 0 \text{ or } x = 10 \text{ or } x = -4}$$

14. Given that $x = 3$ is a zero of the polynomial $p(x) = x^3 - 27$, find all the other zeros, real or complex, of $p(x)$.

3	1	0	0	−27
	↓	3	9	27
	1	3	9	0

$$x^2 + 3x + 9 = 0$$

$$x = \frac{-3 \pm \sqrt{9 - 4(1)(9)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{-27}}{2} = \boxed{\frac{-3 \pm 3\sqrt{3}\,i}{2}}$$

15. Answer TRUE or FALSE for the statements that follow.

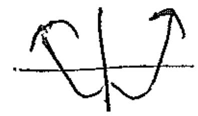
(a) Degree 5 polynomials could have either 4, 2, or 0 turning points. True

(b) Degree 4 polynomials could have either 4, 2, or 0 turning poiints. False

(c) The y-intercept of a polynomial is also known as its constant term. True

(d) Odd degree polynomials are one-to-one functions. False

(e) Even degree polynomials are one-to-one functions. False



16. The model $P(t) = 4.6386e^{0.6642t}$ estimates the number of daffodils that bloom in a certain neighborhood between the years 1980-1986, where t is the number of years since 1980. How many daffodils were there in the year 1985? Round to the nearest daffodil.

$P(5) = 4.6386e^{.6642(5)}$

$= 128.43367157$

≈ 128 daffodils