Name: Key
Recitation Instructor:
Recitation Day and Time:

Studio College Algebra – Final Exam, December 2015

Directions: You will find 28 problems listed below. Each problem is worth 5 points. No notes/books/friends are allowed. Graphing calculator models above the level of a TI-84 plus are not allowed (in particular, calculators with a built in CAS and/or QWERTY keyboard are not allowed). You have one hour to complete this exam. SHOW ALL WORK!

$x_1\ y_1 \quad x_2\ y_2$

1. Find the slope of the line passing through (-2,5) and (4,3). Then, find the equation of the line passing through these two points, presenting your answer in either point-slope or slope-intercept form.

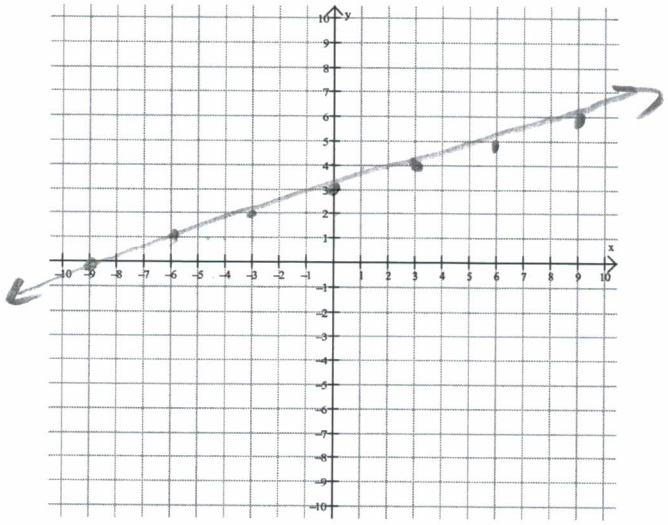
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-5}{4-(-2)} = \frac{-2}{6} = -\frac{1}{3}$$

$$y - 3 = -\frac{1}{3}(x-4) \quad \text{(point slope form)}$$

2. Graph $-x + 3y = 9$ on the grid below. Include all intercepts.

$3y = x + 9$

$y = \frac{1}{3}x + 3$



3. Solve $2x - 7 = |2x + 1|$ and check your answers.

$2x-7=2x+1$ or $-(2x-7)=2x+1$

$0=8$

$\times$

no solution from this part

$-2x+7=2x+1$

$6=4x$

$x=\frac{3}{2}$

Check: $2(\frac{3}{2})-7=-4$ $\times$

This doesn't work either

No Solution

4. Solve $|6x - 7| < 11$.

$-11<6x-7<11$

$-11<6x-7$ and $6x-7<11$

$-4<6x$ and $6x<18$

$-\frac{2}{3}<x$ and $x<3$

$-\frac{2}{3}<x<3$

5. Suppose the cost function in U.S. dollars for x units of a certain product is given by $C(x) = 5x+4000$, and similarly, the revenue function for the product is given by $R(x) = 35x-2000$. How many units must be sold to earn \$3,000 in profit?

Profit = Revenue − cost

$P(x) = 35x - 2000 - (5x + 4000)$

$P(x) = 30x - 6000$

$3000 = 30x - 6000$

$9000 = 30x$

$x = 300$ units

6. (6 points) In a controlled lab environment, some organisms exhibit constant growth over a specific time period. Suppose a certain organism starts out weighing 2 mg, and grows to 8 mg over a 24 hour time period. Find a linear model (use function notation!) that describes the growth of the organism for $0 \leq t \leq 24$ hours.

(0, 2mg)

(24, 8mg)

$\frac{8-2}{24-0} = \frac{6}{24} = \frac{1}{4}$ mg/4

$f(t) = \frac{1}{4}t + 2$ $\quad 0 \leq t \leq 24$

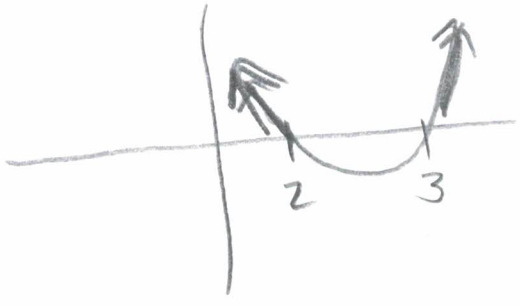
7. The weekly profit function for a business is $P(x) = 40x - 500$, where x is the number of customers. How many more customers must the business add if it wants to increase profits by \$2400 per week?(Hint: Marginal profit)

$\frac{2400}{40} =$ 60 customers

(each customer brings in \$40)

8. Solve the quadratic inequality $x^2 - 5x + 6 > 0$.

$(x-3)(x-2) > 0$



$x < 2$ or $x > 3$

9. Given $f(x) = 7x - 3$, find $f^{-1}(x)$.

$y = 7x - 3$

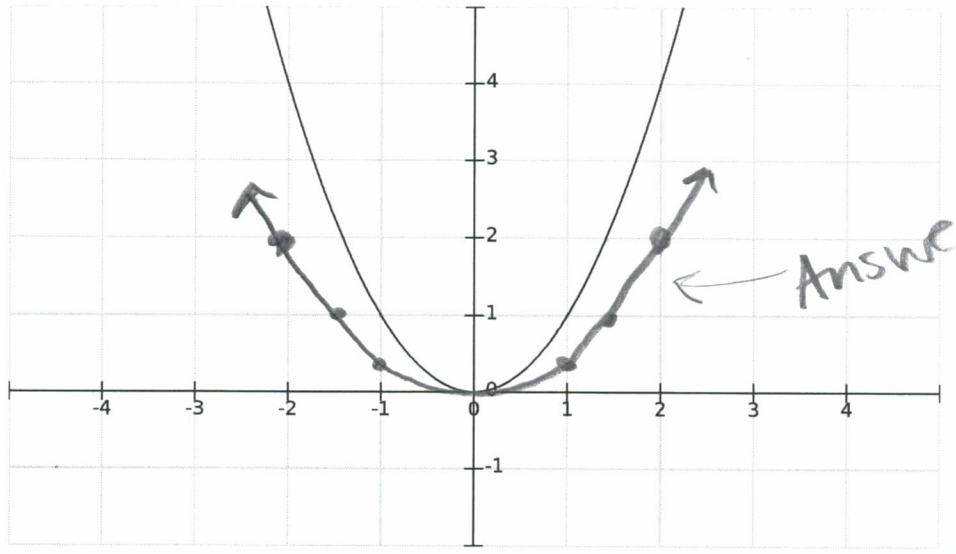
$y + 3 = 7x$

$\frac{y+3}{7} = x$

Answer:

$f^{-1}(x) = \frac{x+3}{7}$

10. Given the graph of $f(x)$ below, graph $\frac{1}{2}f(x)$.



11. Solve $t^2 - 4t - 10 = 12$.

$t^2 - 4t - 22 = 0$

$a = 1$
$b = -4$
$c = -22$

$$t = \frac{4 \pm \sqrt{16 - 4(1)(-22)}}{2(1)}$$

$$t = \frac{4 \pm \sqrt{16 + 88}}{2} = \frac{4 \pm \sqrt{104}}{2}$$

$$= \frac{2 \pm 2\sqrt{26}}{2}$$

$$= \boxed{1 \pm \sqrt{26}}$$

12. The height of a projectile in the air off the ground in meters, t seconds after it is thrown, is given by the equation $s(t) = -4.9t^2 + 12t + 100$. When does the ball reach a maximum height?

(h, k) : vertex

$$h = \frac{-b}{2a}$$

$$h = \frac{-12}{2(-4.9)} = \frac{12}{9.8}$$

$h \approx \boxed{1.22 \text{ seconds}}$

highest point

13. Given $h(x) = e^x + 4$ and $k(x) = \ln(x-4)$, find $k(h(x))$ and $h(k(x))$.

$k(h(x)) = k(e^x+4) = \ln(e^x+4-4)$

$= \ln(e^x) = x \ln(e)$

$= \boxed{x}$

$h(k(x)) = h(\ln(x-4)) = e^{\ln(x-4)} + 4$

$= x - 4 + 4 = \boxed{x}$

14. Solve and check: $8 - x = \sqrt{x+4}$

$64 - 16x + x^2 = x + 4$

$60 - 17x + x^2 = 0$

$(x-12)(x-5) = 0$

$x = 12$ or $x = 5$

$x = 12$ doesn't work (why?)

$\boxed{x = 5 \text{ works}}$

$8 - 5 = \sqrt{5+4}$ ✓

15. If $\log(a) = 1.6$ and $\log(b) = 2.4$, find $\log(ab^2)$.

$$\log(ab^2) = \log(a) + \log(b^2)$$
$$= \log(a) + 2\log(b)$$
$$= 1.6 + 2(2.4)$$
$$= 1.6 + 4.8$$
$$= \boxed{6.4}$$

16. What lump sum would need to be invested at an annual interest rate of 2%, under daily compounding, for 6 years, in order to end up with \$3400? Round answer to the nearest cent.

$$3400 = PV\left(1 + \frac{.02}{365}\right)^{365(6)}$$

$$PV = \frac{3400}{\left[\left(1 + \frac{.02}{365}\right)^{365(6)}\right]}$$

Approx. \$3015.54

17. Solve $4\ln(6x-5)+1=11$. Leave answer exact, i.e., do not use calculator.

$4\ln(6x-5)=10$

$\ln(6x-5)=\frac{10}{4}$

$e^{5/2}=6x-5$

$\frac{e^{5/2}+5}{6}=x$

leave as is

18. Find the domain of $f(x)=\ln(60-7x)$.

$60-7x>0$

$-7x>-60$

$x<\frac{60}{7}$

19. Find 2 different fourth degree polynomials, each having single roots at $x = 3$, $x = 4$ and a double root at $x = -1$. Do not multiply your answers out.

Answer 1: $(x-3)(x-4)(x+1)^2$

Answer 2: $5(x-3)(x-4)(x+1)^2$

other answers possible (why?)

20. Given that $x = -4$ is a zero of the polynomial $p(x) = x^3 + 64$, find all the other zeros, real or complex, of $p(x)$.

$$\begin{array}{r|cccc} -4 & 1 & 0 & 0 & 64 \\ & \downarrow & -4 & 16 & -64 \\ \hline & 1 & -4 & 16 & 0 \end{array}$$

Solve $x^2 - 4x + 16 = 0$

$$x = \frac{4 \pm \sqrt{16 - 4(1)(16)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{-48}}{2} = \frac{4}{2} \pm \frac{4\sqrt{3}\,i}{2}$$

$2 \pm 2\sqrt{3}\,i$

21. For each of the following exponential functions, write down if the function represents 'growth' or 'decay.'

(a) $y = 0.5^{-x}$ growth

(b) $y = 5^{x}$ growth

(c) $y = \left(\frac{4}{3}\right)^{-x}$ decay

(d) $y = \left(\frac{1}{2}\right)^{-x}$ growth

(e) $y = \left(\frac{5}{3}\right)^{x}$ growth

22. Consider the rational function $r(x) = \dfrac{(7x+5)(x-1)}{x^2-8x+12}$.

(a) Find the vertical asymptotes of $r(x)$.

$(x-6)(x-2) = 0$

$x = 6, \; x = 2$

(b) Find the zeros of $r(x)$.

$x = -\frac{5}{7}, \; x = 1$

(c) Find the y-intercept of $r(x)$.

$r(0) = \frac{-5}{12}$

$\left(0, -\frac{5}{12}\right)$

23. Light roast coffee beans cost \$7.00/lb, while dark roast ones cost \$5.50/lb. How much of each type of coffee is needed to create 4.50 pounds of a mixture that costs \$6.75 per pound?

See Fall 2014 Solutions.

24. (a) Write the augmented matrix determined by the following system:

$$4x + 5y = 4$$
$$2x - 3y = 9$$

$$\left[\begin{array}{cc|c} 4 & 5 & 4 \\ 2 & -3 & 9 \end{array}\right]$$

(b) Solve the above system completely by hand (in other words, do not use a calculator).

$4x+5y=4$
$-4x+6y=-18$
$11y=-14$

$y=\frac{-14}{11}$

$2x+\frac{42}{11}=\frac{99}{11}$

$2x=\frac{57}{11}$

$x=\frac{57}{22}$

25. Suppose A is a 4x3 matrix, B is a 3x3 matrix, and C is a 4x4 matrix. Also, assume that all these matrices have real valued entries. Beside each of the following, write down the size of the resulting matrix. If undefined, write down 'undefined.'

(a) $2A$ 4x3

(b) $A+B$ undefined

(c) AB 4x3

(d) BA undefined

(e) $6B$ 3x3

26. Let I_3 denote the 3x3 identity matrix. Find $(7I_3 + 2I_3)(2I_3)$.

$$\left[\begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}\right]\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 18 & 0 & 0 \\ 0 & 18 & 0 \\ 0 & 0 & 18 \end{bmatrix}$$

or $(9I_3)(2I_3) = 18I_3 =$

27. Solve the following rational equation: $\frac{2x+14}{4x+7} = \frac{x+8}{2x+4}$.

$$\frac{(2x+14)(2x+4) - (x+8)(4x+7)}{(4x+7)(2x+4)} = 0.$$

$$\frac{4x^2+8x+28x+56 - (4x^2+7x+32x+56)}{(4x+7)(2x+4)} = 0$$

$$\frac{-3x = 0}{(4x+7)(2x+4)}$$

$$-3x = 0$$

$$\boxed{x = 0}.$$

28. Is it possible to find the inverse of the following matrix? Why or why not? Briefly explain.

$$\begin{matrix} a \\ c \end{matrix}\begin{pmatrix} 1 & 4 \\ -2 & -8 \end{pmatrix}\begin{matrix} b \\ d \end{matrix}$$

$$ad - bc = 1(-8) - 4(-2)$$

$$= -8 + 8$$

$$= 0.$$

No, determinant is zero.

(studio # 10 went over this lab Concept)