Name: Key
Recitation Instructor:
Recitation Day and Time:

Studio College Algebra – Exam 2 – October 2016

Directions: You will find 16 problems listed below. SHOW ALL WORK!! Each problem is worth 5 points. No notes/books/friends are allowed. Graphing calculator models above the level of a TI-84 plus are not allowed (in particular, calculators with a built in CAS and/or QWERTY keyboard are not allowed). You have one hour to complete this exam.

1. Solve $x^2 - 19x + 70 = 0$.

$(x-14)(x-5) = 0$

$\boxed{x = 14 \text{ or } x = 5}$

OR using $a=1, b=-19, c=70$,

$$x = \frac{19 \pm \sqrt{(-19)^2 - 4(1)(70)}}{2(1)}$$

$$x = \frac{19 \pm \sqrt{81}}{2} = \frac{19 \pm 9}{2}; \quad x = \frac{19+9}{2} = 14; \quad x = \frac{19-9}{2} = 5.$$

2. Write $x^2 - 10x + 5$ in the form $a(x-h)^2 + k$.

$x^2 - 10x + 5 = x^2 - 10x + 0 + 5$

$= x^2 - 10x + 25 - 25 + 5$

$= \boxed{(x-5)^2 - 20}$

OR: $h = \frac{-b}{2a}$; $h = \frac{-(-10)}{2(1)} = 5$; $k = 5^2 - 10(5) + 5 = -20$

$\Rightarrow 1(x-5)^2 - 20.$

3. A parabola has vertex at $(3, -4)$ and passes through the point $(1, 7)$. What is the equation of the parabola? Write your answer in the form $y = a(x - h)^2 + k$ (DO NOT MULTIPLY OUT).

$y = a(x-h)^2 + k$

$7 = a(1-3)^2 - 4$

$7 = a(4) - 4$

$11 = 4a$

$a = \frac{11}{4}$;

$y = \frac{11}{4}(x-3)^2 - 4$

4. The height of a ball in the air off the ground in meters, t seconds after it is thrown, is given by the equation $s(t) = -4.9t^2 + 12t + 20$. Answer the questions that follow.)

(a) Find $s(0)$ and describe its practical meaning.

$S(0) = 20$

At time $t = 0$, the inital height of the ball is 20 meters.

(b) What is the practical meaning of solving $s(t) = 0$? (DO NOT SOLVE!).

This solution represents when ball hits the ground.

5. Given $h(x) = 14 - x^2$ and $k(x) = x^2 - 7x$, compute $k(x) - h(x)$.

$$k(x) - h(x) = x^2 - 7x - (14 - x^2)$$
$$= x^2 - 7x - 14 + x^2$$
$$= \underline{2x^2 - 7x - 14}$$

6. Given $r(x) = 3x - 10$ and $m(x) = x^3 - 4x$, compute $r(x)m(x)$.

$$r(x)m(x) = (3x - 10)(x^3 - 4x)$$
$$= 3x^4 - 12x^2 - 10x^3 + 40x$$

7. Consider the functions, $f(x) = x - 5$ and $g(x) = x + 8$:

(a) Using the functions above, find $f(-2) + g(-2)$.

$f(-2) + g(-2) = (-7) + (6) = \boxed{-1}$

(b) Using the functions above, find $f(g(f(4)))$.

$f(g(f(4))) = f(g(-1))$

$= f(7)$

$= \boxed{2}$

8. Solve the quadratic inequality $x^2 - 3x < 18$. (Hint: Use either a case analysis, graphing, or number line method discussed in lecture.)

$x^2 - 3x - 18 < 0$

$(x-6)(x+3) < 0$

$x-6 > 0$ AND $x+3 < 0$

$x > 6$ AND $x < -3$

(No such x exists)

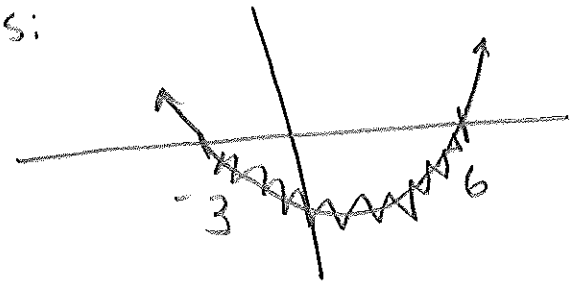
OR

$x-6 < 0$ AND $x+3 > 0$

$x < 6$ AND $x > -3$

$\boxed{-3 < x < 6}$ Solution

With Graphs:



Number line method also fine.

Brian

9. Given $g(t) = \dfrac{t+5}{t}$, find $g^{-1}(t)$.

$y = \dfrac{t+5}{t}$

Solve for t:

$ty = t+5$

$ty - t = 5$

$t(y-1) = 5$

$t = \dfrac{5}{y-1}$

$\boxed{g^{-1}(t) = \dfrac{5}{t-1}}$

10. Solve and check: $x - 6 = \sqrt{x}$

$x^2 - 12x + 36 = x$

$x^2 - 13x + 36 = 0$

$(x-9)(x-4) = 0$

Check $x = 9$:

$9 - 6 = \sqrt{9}$ ✓

$3 = 3$

Check $x = 4$:

$4 - 6 \neq \sqrt{4}$

$-2 \neq 2$.

$\boxed{\text{Only } x = 9}$

11. The profit function for selling x units of a certain product is given by $P(x) = -x^2+900x-160,000$. Answer the following questions.

(a) For what number of units is maximum profit achieved? Include appropriate units in your answer.

Find vertex: $h = -\frac{b}{2a}$; $h = \frac{-900}{2(-1)} = 450$

450 units

(b) What is the maximum profit in this situation? Remember to include appropriate labels (units) in your answer.

$k = P(450) = -450^2 + 900(450) - 160000$

$= \$42500$

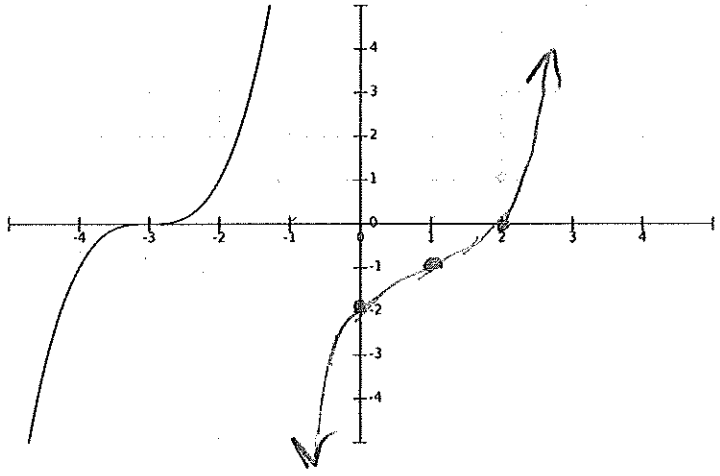
12. A 3-dimensional cartoon portrays an expanding sphere that grows in volume according to the function $V(r) = \frac{4}{3}\pi r^3$, where r is the radius of the sphere, in millimeters. If the radius grows according to the function $r(t) = 3t$, where t is measured in seconds, find $V(r(t))$.

$V(r(t)) = V(3t)$

$= \frac{4}{3}\pi(3t)^3 = \frac{4}{3}\pi(27t^3)$

$= 36\pi t^3$

13. Given the graph of $f(x)$ below, graph $f(x-4)-1$.



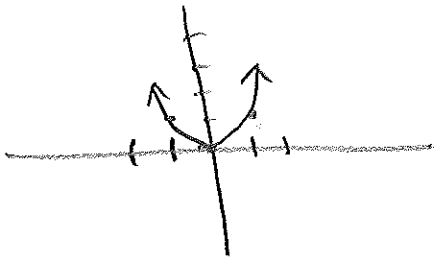
14. Insect resting metabolic rate (RMR) has been found to be scaled positively with body mass (M) according to the equation $RMR = 4.14(M^{0.66})$, where M is measured in mg and RMR is measured in mm^3O_2 per hour. Find the RMR of an insect weighing 1.4 grams.

$RMR = 4.14(1400^{.66})$ Conver g to mg.

$= 493.679 \ mm^3O_2/hr$

15. Consider the function $f(x) = x^2$. Is this a one-to-one function? Explain your reasoning clearly.

No; the graph doesn't pass the HORIZONTAL LINE TEST.



16. Consider the following piecewise function. Write TRUE or FALSE beside each of the statements given below.

$$f(x) = \begin{cases} 1, & x \leq -3 \\ x^3 + 1, & -3 < x \leq 2 \\ -x - 6, & x > 2 \end{cases}$$

(a) $f(2) = 9$. True

(b) $f(2) = -4$. False

(c) $f(-3) = 1$. True

(d) $f(-3) = -26$. False

(e) $f(-3) = -9$. False