Name: Key
Recitation Instructor:
Recitation Day and Time:

Studio College Algebra – Exam 3 – Fall 2016

Directions: You will find 16 problems listed below. Each problem is worth 5 points. No notes/books/friends are allowed. Graphing calculator models above the level of a TI-84 plus are not allowed (in particular, calculators with a built in CAS and/or QWERTY keyboard are not allowed). You have one hour to complete this exam. SHOW ALL WORK!

1. Rewrite the formula by taking the logarithm of both sides, and then use properties of logarithms wherever applicable: $y = \frac{8}{x^2}$ (you may assume $x > 0$.)

$\ln y = \ln 8 - 2 \ln x$

2. Rewrite the formula by taking the logarithm of both sides, and then use properties of logarithms wherever applicable: $y = (1.4)7^x$.

$\ln y = \ln(1.4) + \ln(7^x)$

$\ln y = \ln(1.4) + x \ln(7)$

3. If $\log(a) = 2.4$ and $\log(b) = 1.2$, find $\log\left(\frac{a^3}{\sqrt[7]{b}}\right)$.

$\log(a^3) - \log(b^{1/7})$

$= 3\log(a) - \frac{1}{7}\log(b)$

$= 3(2.4) - \frac{1}{7}(1.2)$

$= \boxed{\frac{246}{35}}$ or ≈ 7.02857

4. Roughly what lump sum would need to be invested at an annual interest rate of 2%, under continuous compounding, for 5 years, in order to end up with \$10000? Round answer to the nearest cent.

$FV = Pe^{rt}$

$10000 = PV\left(e^{.02(5)}\right)$

$\frac{10000}{e^{.02(5)}} = PV$

$PV \approx 9048.37$

5. Solve $6^{x-7} = 8$. Leave answer exact, i.e., do not use calculator.

$$\ln 6^{x-7} = \ln 8$$

$$(x-7)\ln 6 = \ln 8$$

$$x - 7 = \frac{\ln(8)}{\ln 6}.$$

$$\boxed{x = 7 + \frac{\ln(8)}{\ln(6)}}$$

6. Solve $6\ln(2x-7) - 1 = 23$. Leave answer exact, i.e., do not use calculator.

$$6\ln(2x-7) = 24$$

$$\ln(2x-7) = 4$$

$$e^4 = 2x - 7$$

$$\boxed{\frac{e^4+7}{2} = x}$$

7. Given $f(x) = \log_3(x+1)$, find $f^{-1}(x)$.

$y = \log_3(x+1)$

$3^y = x+1$

$3^y - 1 = x$

$f^{-1}(x) = 3^x - 1$

8. Find the domain of $f(x) = 37\ln(6-4x) + 1729$.

$6 - 4x > 0$

$-4x > -6$

$x < \frac{3}{2}$

9. The function $P(t) = 21.109 - 5.686\ln(t+1)$ describes the revenue, in thousands of dollars, for the sale of a product t weeks after an ad campaign for the product ended, where $0 \le t \le 10$. Find $P(4)$, round to the nearest cent, and interpret the meaning of $P(4)$ in a complete sentence.

$P(4) = 21.109 - 5.686\ln(4+1)$

$= 11.95774$

4 weeks after the ad campaign ended, \$11,957.74 was the revenue generated.

10. What are all the real and complex zeros of $x^3 + 125$, given that one zero is $x = -5$?

$$\begin{array}{r|cccc} -5 & 1 & 0 & 0 & 125 \\ & \downarrow & -5 & 25 & -125 \\ \hline & 1 & -5 & 25 & 0 \end{array}$$

$x^2 - 5x + 25 = 0$

$a = 1,\ b = -5,\ c = 25$:

$$x = \frac{5 \pm \sqrt{25 - 4(1)(25)}}{2(1)} = \frac{5 \pm \sqrt{-75}}{2}$$

$$= \frac{5}{2} \pm \frac{5\sqrt{3}}{2}i \quad \& \quad x = -5$$

11. What is the horizontal asymptote of the function $f(x) = e^x + 7$? Explain briefly how you arrived at your answer, using proper math vocabulary and grammar.

$y = 7$

The graph of $y = e^x$ has horizontal asymptote $y = 0$; $f(x)$ has the shape of $y = e^x$ but is vertically shifted 7 units, therefore $y = 7$ is the horizontal asymptote of $f(x)$.

12. Find 2 different fourth degree polynomials, each having single roots at $x = 9$, $x = 7$ and a double root at $x = -4$. DO NOT multiply your answers out.

Answer 1: $p(x) = (x-9)(x-7)(x+4)^2$

Answer 2: $p(x) = k(x-9)(x-7)(x+4)^2$

k is a real # other than 0 or 1.

13. Given the revenue function $R(x) = 225x - x^3$, where x is a number of units, what numbers of units give zero revenue?

$0 = 225x - x^3$

$0 = x(225 - x^2)$

$0 = x(15-x)(15+x)$

$x = 0$ or $x = 15$ units

(ignore $x = -15$)

14. Given that $x = -2$ is a zero of the polynomial $p(x) = x^3 + 6x^2 + 21x + 26$, find all the other zeros, real or complex, of $p(x)$.

$$\begin{array}{r|cccc} -2 & 1 & 6 & 21 & 26 \\ & \downarrow & -2 & -8 & -26 \\ \hline & 1 & 4 & 13 & 0 \end{array}$$

$$x = \frac{-4 \pm \sqrt{16 - 4(1)(13)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{-36}}{2} = \frac{-4 \pm 6i}{2} = -2 \pm 3i$$

15. Answer TRUE or FALSE for the statements that follow. (If you answer TRUE, it means the statement is always true!)

(a) Degree n polynomials, where n is even, are one-to-one functions. False

(b) Degree n polynomials, where n is odd, are one-to-one functions. False

(c) Quadratic functions are also called degree 2 polynomials. True

(d) The constant term of a polynomial and it's y-intercept are the same. True

(e) $f(x) = \left(\frac{1}{4}\right)^{-2x}$ is an exponential growth function. True

16. We discussed the general form of an exponential function in lecture: $g(x) = a^x$, where a is the base with $a > 0$ and $a \neq 1$. Answer the following questions:

(a) For what values of a does $g(x)$ represent exponential growth?

$a > 1$

(b) For what values of a does $g(x)$ represent exponential decay?

$0 < a < 1$.