Name: Key
Recitation Instructor:
Recitation Day and Time:

Studio College Algebra – Exam 3 – April 2016

Directions: You will find 16 problems listed below. Each problem is worth 5 points. No notes/books/friends are allowed. Graphing calculator models above the level of a TI-84 plus are not allowed (in particular, calculators with a built in CAS and/or QWERTY keyboard are not allowed). You have one hour to complete this exam. SHOW ALL WORK!

1. Rewrite the following in exponential form: $\ln(x+2) = 7$. Do not use your calculator.

$$e^7 = x + 2$$

2. Rewrite the following in logarithmic form: $3^{-5} = \frac{1}{243}$. Do not use your calculator.

$$\log_3\left(\frac{1}{243}\right) = -5$$

3. If $\log(a) = 2.1$ and $\log(b) = 1.5$, find $\log\left(\frac{a^2}{\sqrt[3]{b}}\right)$.

$$\log\left(\frac{a^2}{\sqrt[3]{b}}\right) = \log(a^2) - \log(\sqrt[3]{b})$$

$$= 2\log(a) - \frac{1}{3}\log(b)$$

$$= 2(2.1) - \frac{1}{3}(1.5)$$

$$= 4.2 - .5$$

$$= \boxed{3.7}$$

4. What lump sum would need to be invested at an annual interest rate of 2%, under continuous compounding, for 4 years, in order to end up with \$9000? Round answer to the nearest cent.

$$FV = PV(e^{rt})$$

$$9000 = PV(e^{.02(4)})$$

$$\frac{9000}{(e^{.08})} = PV$$

$$PV = \$8308.05$$

5. Solve $3^{(x-2)} = 5$. Leave answer exact, i.e., do not use calculator.

$$\log_3(5) = x - 2.$$

$$\boxed{\log_3(5) + 2 = x}$$

OR $\dfrac{\log(5)}{\log(3)} + 2 = x$

OR $\dfrac{\ln(5)}{\ln(3)} + 2 = x$

other common answers from other valid methods.

6. Solve $3\ln(x+4) - 1 = 8$. Leave answer exact, i.e., do not use calculator.

$$3\ln(x+4) = 9$$

$$\ln(x+4) = 3$$

$$e^3 = x + 4$$

$$\boxed{e^3 - 4 = x}$$

7. Given $f(x) = mx + b$, where $m \neq 0$, find $f^{-1}(x)$. (Hint: m and b are parameters here, so you can use the usual process of finding $f^{-1}(x)$ as discussed in lecture.)

Step 1: $y = mx + b, \quad m \neq 0$

Step 2: $y - b = mx, \quad m \neq 0$

$\frac{y-b}{m} = x, \quad m \neq 0.$

$\boxed{f^{-1}(x) = \frac{x-b}{m}}$ Answer

8. Find the domain of $f(x) = \log(4x - 29)$.

$4x - 29 > 0.$

$4x > 29$

$\boxed{x > \frac{29}{4}}$

Blane

9. The function $P(t) = 21.109 - 5.686\ln(t+1)$ describes the revenue, in thousands of dollars, for the sale of a product t weeks after an ad campaign for the product ended, where $0 \le t \le 10$. Find $P(4)$, round to the nearest cent, and interpret the meaning of $P(4)$ in a complete sentence.

$$P(4) = 21.109 - 5.686 \ln(4+1)$$

$$= 11.95774$$

4 weeks after the ad campaign ended, \$11,957.74 was brought in.

10. What are all the real and complex zeros of $x^3 - 64$, given that one zero is $x = 4$?

$$\begin{array}{r|rrrr} 4 & 1 & 0 & 0 & -64 \\ & \downarrow & 4 & 16 & 64 \\ \hline & 1 & 4 & 16 & 0 \end{array}$$

$$x^2 + 4x + 16 = 0$$

$$x = \frac{-4 \pm \sqrt{16 - 4(1)(16)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{-48}}{2} = \frac{-4 \pm 4\sqrt{3}\,i}{2} = -2 \pm 2\sqrt{3}\,i$$

along with $x = 4$

Adelina

11. What is the horizontal asympote of $f(x) = 2^x - 5$? Explain briefly how you arrived at your answer, using proper math vocabulary and grammar.

Answer: $y = -5$.

The graph of $f(x)$ is the graph of $y = 2^x$ shifted down 5 units; hence the asymptote of $y = 0$ on $y = 2^x$ shifts down 5 units as well.

12. Find a fourth degree polynomial having single roots at $x = -2$ and $x = 3$, and a double root at $x = -5$. Do not multiply your answer out.

$$p(x) = (x+2)(x-3)(x+5)^2$$

Demara.

13. Given the revenue function $R(x) = 289x - x^3$, where x is a number of units, what numbers of units give zero revenue?

$$0 = 289x - x^3$$
$$0 = (289 - x^2)(x)$$
$$0 = (17 - x)(17 + x)(x)$$

$x = 0$ units or $x = 17$ units

(ignore $x = -17$ units)

14. Given that $x = 3$ and $x = -3$ are zeros of the polynomial $p(x) = x^4 - 2x^3 - 7x^2 + 18x - 18$, find all the other zeros, real or complex, of $p(x)$.

$$\begin{array}{r|rrrrr} 3 & 1 & -2 & -7 & 18 & -18 \\ & \downarrow & 3 & 3 & -12 & 18 \\ \hline -3 & 1 & 1 & -4 & 6 & 0 \\ & \downarrow & -3 & 6 & -6 & \\ \hline & 1 & -2 & 2 & 0 & \end{array}$$

$$x^2 - 2x + 2 = 0.$$

$$x = \frac{2 \pm \sqrt{4 - 4(1)(2)}}{2(1)} = \frac{2 \pm \sqrt{-4}}{2}$$

$$= \frac{2 \pm 2i}{2}$$

$$= \boxed{1 \pm i}.$$

Lee

15. For each statement below, answer "S" if the statement is sometimes true, "A" if the statement is always true, or "N" if the statement is never true. If the statement is about polynomials, you can assume coefficients of the polynomial are real valued.

S (a) Linear functions are one-to-one functions. (not lines with zero slope)

N (b) Degree n polynomials, where n is an even number greater than or equal to 2, are one-to-one functions.

N (c) Quadratic functions are one-to-one functions. (same as (b)).

N (d) $f(x) = k$, where k is a real number, is a one to one function. these are lines with zero slope

A (e) The constant term of a polynomial is the same as its y-intercept.

16. We discussed the general form of an exponential function in lecture: $g(x) = a^x$, where a is the base with $a > 0$ and $a \neq 1$. Answer the following questions:

(a) For what values of a does $g(x)$ represent exponential growth?

$a > 1$

(b) For what values of a does $g(x)$ represent exponential decay?

$0 < a < 1$.