Name: (Key)
Recitation Instructor:
Recitation Day and Time:

Studio College Algebra – Final Exam – Spring 2016

Directions: You will find 28 problems listed below. Each problem is worth 5 points. No notes/books/friends are allowed. Graphing calculator models above the level of a TI-84 plus are not allowed (in particular, calculators with a built in CAS and/or QWERTY keyboard are not allowed). You have one hour to complete this exam. SHOW ALL WORK!

1. Find the equation of the line passing through (1,3) and (5,9). Present answer in slope-intercept form.

$$m = \frac{9-3}{5-1} = \frac{6}{4} = \frac{3}{2}$$

$$y - y_0 = m(x - x_0)$$

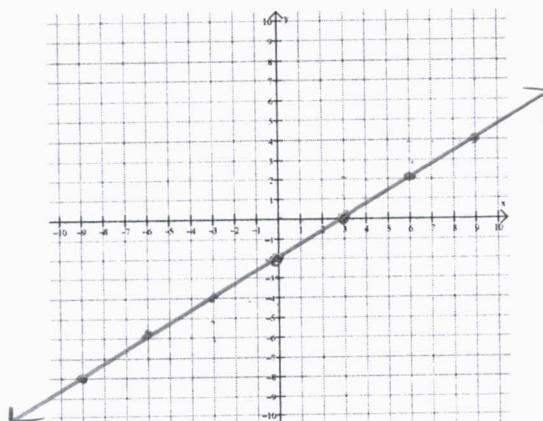$$y - 3 = \frac{3}{2}(x-1); \quad y-3 = \frac{3}{2}x - \frac{3}{2};$$

Slope-Int Form $\boxed{y = \frac{3}{2}x + \frac{3}{2}}$

2. Graph $2x - 3y = 6$ on the grid below. Include all intercepts.

$$-3y = -2x + 6$$
$$y = \frac{2}{3}x - 2$$

3. Solve $|2x + 5| = 3x - 7$ and check your answers.

$$2x + 5 = 3x - 7 \quad \text{or} \quad 2x + 5 = -3x + 7$$

$$12 = x \quad \text{or} \quad 5x = 2; \quad x = \frac{2}{5}$$

Check $x = 12$:

Left Side: $|24 + 5| = 29$ ✓

Right Side: $3(12) - 7 = 29$

Check $x = \frac{2}{5}$

Left: $|\frac{4}{5} + \frac{25}{5}| = \frac{29}{5}$ ✗

Right: $\frac{6}{5} - \frac{35}{5} = -\frac{29}{5}$

Only $x = 12$ works

4. Solve $|8x - 11| < 6$.

$$-6 < 8x - 11 < 6$$

$-6 < 8x - 11 \quad$ and $\quad 8x - 11 < 6$

$5 < 8x \quad\quad\quad$ and $\quad 8x < 17$

$\frac{5}{8} < x \quad\quad\quad$ and $\quad x < \frac{17}{8}$

$$\frac{5}{8} < x < \frac{17}{8}$$

5. Suppose the cost function for a certain product is given by $C(x) = 12x + 2000$ and the revenue function for the product is given by $R(x) = 10x + 3000$. What is the profit function for this situation?

$$P(x) = R(x) - C(x)$$
$$= 10x + 3000 - (12x + 2000)$$
$$= -2x + 1000$$

$$\boxed{P(x) = -2x + 1000}$$

6. A truck depreciates in value according to a linear model. If the initial value of the truck is $24,000, and the value 40 years later is $0, what was the depreciated value of the truck after 7 years?

$(0, \$24000)$

$(40, 0)$

$\text{Slope} = \dfrac{-24000}{40} = \$-600/\text{yr}$

$y = -600x + 24000$

Plug in $x = 7$; $-600(7) + 24000$
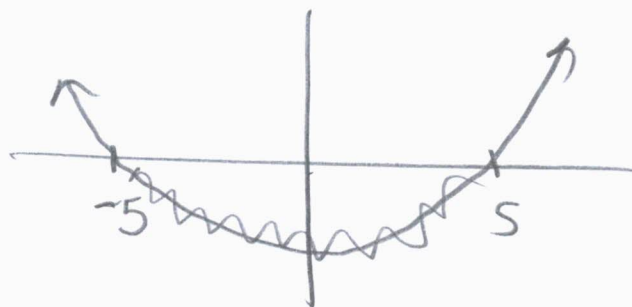
$$= \boxed{\$19800}$$

7. The weekly profit function for a business is $P(x) = 10x - 300$, where x is the number of customers. How many more customers must the business add if it wants to increase profits by $400 per week?

$$400/10 = 40$$

$$\boxed{40 \text{ customers}}$$

8. Solve the quadratic inequality $x^2 - 25 < 0$.

$$(x-5)(x+5) < 0$$

$$\boxed{-5 < x < 5}$$

9. Given $f(x) = \dfrac{7x - 11}{23}$, find $f^{-1}(x)$.
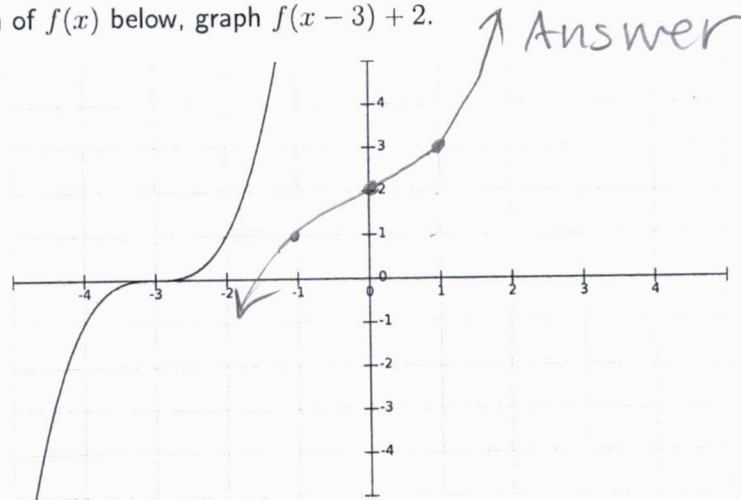
$$y = \frac{7x-11}{23}$$

$$23y = 7x - 11$$

$$23y + 11 = 7x$$

$$\frac{23y+11}{7} = x$$

$$\boxed{f^{-1}(x) = \frac{23x+11}{7}}$$

10. Given the graph of $f(x)$ below, graph $f(x - 3) + 2$.

↗ Answer

11. Solve $t^2 - 5t + 14 = 17$.

$$t^2 - 5t - 3 = 0$$

$$t = \frac{5 \pm \sqrt{25 - 4(1)(-3)}}{2(1)}$$

$$\boxed{t = \frac{5 \pm \sqrt{37}}{2}}$$

12. The height of a ball in the air off the ground in meters, t seconds after it is thrown, is given by the equation $s(t) = -4.9t^2 + 98t + 4$. When does the ball reach a maximum height?

$$h = \frac{-b}{2a}; \quad h = \frac{-98}{2(-4.9)} = \frac{-98}{-9.8} = 10$$

$$\boxed{@ \ t = 10 \ seconds}$$

13. Given $h(x) = x + 2$ and $k(x) = x^2 - 9x$, find $k(h(x))$.

$$K(h(x) = K(x+2)$$
$$= (x+2)^2 - 9(x+2)$$
$$= x^2 + 4x + 4 - 9x - 18$$
$$= x^2 - 5x - 14$$

14. Solve and check: $x + 2 = \sqrt{9x - 2}$

$$x^2 + 4x + 4 = 9x - 2$$
$$x^2 - 5x + 6 = 0$$
$$(x-3)(x-2) = 0$$
$$x = 3 \text{ or } x = 2$$

$x = 3$:
$3 + 2 = 5$;
$\sqrt{27-2} = \sqrt{25} = 5$ ✓

$x = 2$:
$2 + 2 = 4$;
$\sqrt{18-2} = 4$ ✓

Both x=3 and x=2 are solutions

15. If $\log(a) = 2$ and $\log(b) = 3$, find $\log(a\sqrt[3]{b})$.

$$\log\left(a\sqrt[3]{b}\right) = \log(a) + \log\left(\sqrt[3]{b}\right)$$
$$= \log(a) + \frac{1}{3}\log(b)$$
$$= 2 + \frac{1}{3}(3)$$
$$= 2 + 1 = \boxed{3}$$

16. What lump sum would need to be invested at an annual interest rate of 3%, under monthly compounding, for 7 years, in order to end up with $6900? Round answer to the nearest cent.

$$6900 = PV\left(1 + \frac{.03}{12}\right)^{.03(7)}$$

$$\frac{6900}{\left(1 + \frac{.03}{12}\right)^{.03(7)}} = PV$$

$$\boxed{PV = \$6896.38}$$

8

17. Solve $2\ln(3x - 4) + 1 = 11$. Leave answer exact, i.e., do not use calculator.

$$2\ln(3x-4) = 10$$

$$\ln(3x-4) = 5$$

$$e^5 = 3x - 4$$

$$e^5 + 4 = 3x$$

$$\frac{e^5 + 4}{3} = x$$

18. Find the domain of $f(x) = \log(6x - 29)$.

$$6x - 29 > 0$$

$$6x > 29$$

$$x > \frac{29}{6}$$

19. Find 2 different fourth degree polynomials, each having single roots at $x = -53$, $x = 17$ and a double root at $x = 20$. Do not multiply your answers out.

Answer 1: $(x+53)(x-17)(x-20)^2$

Answer 2: $K(x+53)(x-17)(x-20)^2$

$K \in \mathbb{R}, \; K \neq 0, \; K \neq 1.$

20. Given that $x = -5$ is a zero of the polynomial $p(x) = x^3 + 125$, find all the other zeros, real or complex, of $p(x)$.

$$\begin{array}{r|rrrr} -5 & 1 & 0 & 0 & 125 \\ & & -5 & 25 & -125 \\ \hline & 1 & -5 & 25 & 0 \end{array}$$

$x^2 - 5x + 25 = 0$

$x = \dfrac{5 \pm \sqrt{25 - 4(1)(25)}}{2(1)} = \dfrac{5 \pm \sqrt{-75}}{2}$

other zeros

$\dfrac{5 \pm 5\sqrt{3}\,i}{2}$

21. For each of the following exponential functions, write down if the function represents 'growth' or 'decay.'

(a) $y = e^{-x}$ Decay

(b) $y = 3^x$ Growth

(c) $y = \left(\frac{3}{4}\right)^x$ Decay

(d) $y = \left(\frac{2}{5}\right)^{-x}$ Growth

(e) $y = \left(\frac{1}{3}\right)^{-x}$ Growth

22. Consider the rational function $r(x) = \dfrac{(4x + 15)(x + 19)}{x^2 - 7x + 6}$.

(a) Find the vertical asymptotes of $r(x)$.

$$(x-6)(x-1) = 0 \; ; \quad \boxed{x = 6, \; x = 1}$$

(b) Find the horizontal asymptote of $r(x)$.

$$\boxed{y = 4}$$

(c) Find the zeros of $r(x)$.

$$x = -15/4, \quad x = -19$$

(d) Find the y-intercept of $r(x)$.

$$r(0) = \frac{15(19)}{6} = \frac{95}{2}$$

$$\boxed{\left(0, \frac{95}{2}\right)}$$

23. (8 points) Set up a system and solve the following problem. Light roast coffee beans cost $4.00/lb, medium roast coffee beans costs $1.00/lb, and dark roast coffee beans cost $4.50/lb. If there is twice as much medium roast as there is of the light roast, how much of each type of coffee is needed to create 5 pounds of a mixture that costs $3.50 per pound? Be sure to indicate the meaning of any variables used in setting up this problem.

$x = $ amt of light
$y = $ " " med
$z = $ " " dark

$$\begin{cases} x + y + z = 5 \\ 4x + y + 4.5z = 5(3.50) \\ y = 2x \end{cases} \quad = 2x$$

$$\begin{cases} 3x + z = 5 \\ 6x + 4.5z = 17.5 \end{cases}$$

$3x + 3 = 5$
$3x = 2$
$\boxed{x = 2/3 \text{ lbs};}$

$\boxed{y = 4/3 \text{ lbs}}$

$$\begin{cases} -6x - 2z = -10 \\ 6x + 4.5z = 17.5 \end{cases}$$

$2.5z = 7.5;$ $\boxed{z = 3 \text{ lbs};}$

24. (

Solve: $\begin{aligned} 3x + 2y &= 9 \\ 2x + 3y &= 10 \end{aligned}$

$$\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{pmatrix} 9 \\ 10 \end{pmatrix}$$

Using any method you like (except the calculator method), solve the above system.

$$\frac{1}{5}\begin{bmatrix} 3 & -2 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 3/5 & -2/5 \\ -2/5 & 3/5 \end{bmatrix}$$

$$\begin{bmatrix} 3/5 & -2/5 \\ -2/5 & 3/5 \end{bmatrix}\begin{bmatrix} 9 \\ 10 \end{bmatrix} = \begin{bmatrix} 27/5 - 20/5 \\ -18/5 + 30/5 \end{bmatrix} = \begin{bmatrix} 7/5 \\ 12/5 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

12

25. Suppose A is a 4x3 matrix, B is a 3x4 matrix, and C is a 3x3 matrix. Also, assume that all these matrices have real valued entries. Beside each of the following, write down the size of the resulting matrix. If undefined, write down 'undefined.'

(a) AB 4×4

(b) A-B undefined

(c) $(AB)(CB)$ undefined
    4×4 3×4

(d) CB 3×4
    3×3 3×4

(e) C^{-1} 3×3

26. Let I_3 denote the 3x3 identity matrix. Find $(7I_3 + 2I_3)(11I_3)$.

$$\begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} 99 & 0 & 0 \\ 0 & 99 & 0 \\ 0 & 0 & 99 \end{bmatrix}$$

13

27. Solve the following rational equation: $\dfrac{x+7}{x+13} = \dfrac{x-3}{5x-1}$

$$\frac{(x+7)(5x-1) - (x-3)(x+13)}{(x+13)(5x-1)} = 0$$

$$\vdots$$

$$\frac{4x^2 + 24x + 32}{(x+13)(5x-1)} = 0 ;$$

$$4(x^2 + 6x + 8) = 0 ;$$

$\rightarrow$ $4(x+4)(x+2) = 0$

$$\boxed{x = -4 \text{ or } x = -2}$$

28. Find the inverse of the following matrix:

$$\begin{pmatrix} 1 & 5 \\ 4 & -3 \end{pmatrix}$$

$$\frac{1}{-3-20} \begin{bmatrix} -3 & -5 \\ -4 & 1 \end{bmatrix}$$

$$= \frac{1}{-23} \begin{bmatrix} -3 & 5 \\ -4 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} 3/23 & 5/23 \\ 4/23 & -1/23 \end{bmatrix}}$$