Key

Name:
Recitation Instructor:
Recitation Day and Time:

Studio College Algebra – Exam 2 – March 2017

Directions: You will find 16 problems listed below. SHOW ALL WORK!! Each problem is worth 5 points. No notes/books/friends are allowed. Graphing calculator models above the level of a TI-84 plus are not allowed (in particular, calculators with a built in CAS and/or QWERTY keyboard are not allowed). You have one hour to complete this exam.

1. Solve $x^2 + 4x - 12 = 0$.

$(x+6)(x-2) = 0$

$x = -6, 2$

2. Write $x^2 + 6x - 1$ in the form $a(x-h)^2 + k$.

$h = \frac{-b}{2a}$ $\quad h = \frac{-6}{2} = -3$

$k = f(h) = f(-3) = (-3)^2 + 6(-3) - 1$

$= 9 - 18 - 1$

$= -10$

$a = 1$

$x^2 + 6x - 1 = (x-(-3))^2 - 10$

$= (x+3)^2 - 10$

3. A parabola has vertex at $(3, 4)$ and passes through the point $(5, 6)$. What is the equation of the parabola? Write your answer in the form $y = a(x - h)^2 + k$ (DO NOT MULTIPLY OUT).

$y = a(x-3)^2 + 4$ $\qquad$ $y = \frac{1}{2}(x-3)^2 + 4$

$6 = a(5-3)^2 + 4$

$6 = a(4) + 4$

$2 = 4a$

$\frac{1}{2} = a$

4. The height of a ball in the air off the ground in meters, t seconds after it is thrown, is given by the equation $s(t) = -4.9t^2 + 12t + 12$. When does the ball hit the ground? (Hint: When the ball hits the ground, what is the distance off the ground? Use this fact, along with the quadratic formula.)

Solve $s(t) = 0$,

$$\frac{-12 \pm \sqrt{144 - 4(-4.9)(12)}}{-9.8} = t$$

$x_1 = -.763$

$x_2 = 3.212$

Approximately @ 3.212 seconds

5. Given $h(x) = 4x + 5$ and $k(x) = 9x^2 - 2x$, find $k(x) - h(x)$.

$9x^2 - 2x - (4x + 5)$

$9x^2 - 2x - 4x - 5$

$9x^2 - 6x - 5$

6. Given $r(x) = 5x + 1$ and $m(x) = 16x^3 - 2x$, find $r(x)m(x)$.

$(5x + 1)(16x^3 - 2x)$

$80x^4 - 10x^2 + 16x^3 - 2x$

or $80x^4 + 16x^3 - 10x^2 - 2x$; either format is fine.

7. Consider the functions, $f(x) = x + 1$ and $g(x) = 53$:

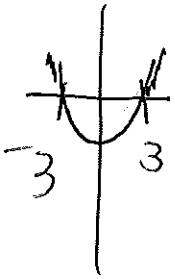
(a) Using the functions above, find $f(2) + g(2)$. $f(2) = 2+1 = 3$

$g(2) = 53$

$= 56$

(b) Using the functions above, find $g(g(g(6000)))$. $= 53$

8. Solve the quadratic inequality $x^2 - 2 > 7$. (Hint: Use either a graphing or number line method discussed in lecture.)



$x^2 - 2 > 7$; $x^2 - 9 > 0$; $(x-3)(x+3) > 0$

$x > 3$ OR $x < -3$

(refer to graph & extract appropriate intervals)

9. Given $f(x) = \dfrac{x-4}{x}$, find $f^{-1}(x)$.

$$y = \frac{x-4}{x}$$

$$xy = x-4$$

$$xy - x = -4$$

$$x(y-1) = -4$$

$$x = \frac{-4}{y-1}$$

$$f^{-1}(x) = \frac{-4}{x-1}$$

10. Solve and check: $x = \sqrt{x+12}$

1pt $x^2 = x+12$

1pt $x^2 - x - 12 = 0$

1pt $(x-4)(x+3) = 0$

1pt $x = 4, -3$

check

$x = -3$

$-3 = \sqrt{-3+12}$

$= \sqrt{9}$

$\neq 3$

$x = 4$

$4 = \sqrt{4+12}$

$= \sqrt{16}$

$= 4$ ✓

Only $x = 4$ works

11. The profit function for selling x units of a certain product is given by $P(x) = -x^2 + 8x - 2$, where $P(x)$ is measured in **thousands**. For what number of units will there be at least $5000 in profit? *Hint: instead of using the number 5000 as part of your calculations, what number should be used?*

$P(x) = -x^2 + 8x - 2$

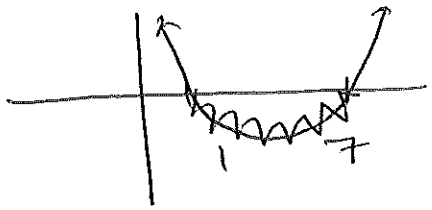
$5 \leq -x^2 + 8x - 2$

$0 \leq -x^2 + 8x - 7$

$x^2 - 8x + 7 \leq 0$

$(x-7)(x-1)$

$x = 7, 1$



$1 \leq x \leq 7$

12. A 3-dimensional cartoon portrays an expanding sphere that grows in volume according to the function $V(r) = \frac{4}{3}\pi r^3$, where r is the radius of the sphere, in millimeters. If the radius grows according to the function $r(t) = 2t$, where t is measured in seconds, find and interpret $V(r(1))$.

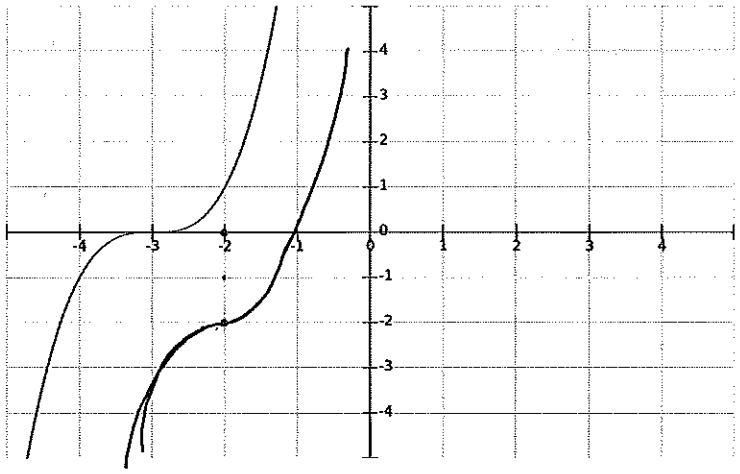
$V(r(t)) = \frac{4}{3}\pi (2t)^3$ 2 pts

$V(r(1)) = \frac{4}{3}\pi (2)^3$ 1 pt

$= \frac{32\pi}{3}\ mm^3$ 1 pt.

This means the volume of the sphere has grown to $\frac{32\pi}{3}\ mm^3$ after 1 sec.

13. Given the graph of $f(x)$ below, graph $f(x-1)-2$.



14. Insect resting metabolic rate (RMR) has been found to be scaled positively with body mass (M) according to the equation $RMR = 4.14(M^{0.66})$, where M is measured in mg and RMR is measured in mm^3O_2 per hour. Find the RMR of an insect weighing 2.7 grams.

$2.7g \times \frac{1000mg}{1g} = 2700mg$

$RMR = 4.14(2700^{.66})$

$= 761.55\ mm^3O_2$

15. A student claims that all lines, excluding vertical lines, are one-to-one functions. Is the student correct? Use examples in your explanation.

no. The constant line $y = 3$ for example is not one to one.

More generally, lines of the form

$f(x) = mx + b$ where $m = 0$

are horizontal, these are not one-to-one.

16. Consider the following piecewise function. Write TRUE or FALSE beside each of the statements given below.

$$f(x) = \begin{cases} 14, & x \leq -3 \\ x^2, & -3 < x \leq 2 \\ -x, & x > 2 \end{cases}$$

(a) $f(2) = -2$. F
(b) $f(2) = 4$. T
(c) $f(-3) = 14$. T
(d) $f(-3) = 3$. F
(e) $f(-3) = 9$. F