Name: Key
Recitation Instructor:
Recitation Day and Time:

Studio College Algebra – Exam 3 – April 2017

Directions: You will find 16 problems listed below. Each problem is worth 5 points. No notes/books/friends are allowed. Graphing calculator models above the level of a TI-84 plus are not allowed (in particular, calculators with a built in CAS and/or QWERTY keyboard are not allowed). You have one hour to complete this exam. SHOW ALL WORK!

1. Rewrite the formula by taking the logarithm of both sides, and then use properties of logarithms wherever applicable: $y = \frac{11}{x^3}$ (you may assume $x > 0$.)

$\log y = \log \frac{11}{x^3}$

$\log y = \log 11 - 3\log x$

$\log_{11} y = 1 - 3\log_{11} x$

Answer in base 11.

* Could also use natural logarithm on this * #2.

2. Rewrite the formula by taking the logarithm of both sides, and then use properties of logarithms wherever applicable: $y = (2.7)5^x$.

$\log y = \log (2.7)(5^x)$

$\log y = \log 2.7 + \log 5^x$

$\log y = \log 2.7 + x \log 5$

$\log_5 y = \log_5 2.7 + x$

Answer in base 5

3. If $\log(a) = 2.6$ and $\log(b) = 1.4$, find $\log\left(\frac{a}{\sqrt[3]{b}}\right)$.

$$\log\left(\frac{a}{\sqrt[3]{b}}\right) = \log a - \frac{1}{3}\log b$$
$$= 2.6 - \frac{1}{3}(1.4)$$
$$= 3.1$$

4. Approximately what lump sum would need to be invested at an annual interest rate of 3%, under continuous compounding, for 5 years, in order to end up with \$5000? Round answer to the nearest cent.

$P = P_0 e^{rt}$

$r = 3\% = 0.03$

$t = 5$ yrs

$P = \$5000$

$P_0 = ?$

$$5000 = P_0 e^{(.03)(5)}$$
$$5000 = P_0 e^{.15}$$
$$\$4303.54 = P_0$$

5. Solve $5^{(x+4)} = 3$. Leave answer exact, i.e., do not use calculator.

$$5^{(x+4)} = 3$$

$$\log_5 5^{x+4} = \log_5 3$$

$$x+4 = \log_5 3$$

$$x = \log_5 3 - 4$$

6. Solve $3\ln(x+2) - 1 = 6$. Leave answer exact, i.e., do not use calculator.

$$3\ln(x+2) = 7$$

$$\ln(x+2) = \frac{7}{3}$$

$$e \qquad\qquad e$$

$$x+2 = e^{7/3}$$

$$x = e^{7/3} - 2$$

7. Given $f(x) = 6x + 4$, find $f^{-1}(x)$.

$$y = 6x + 4$$

$$y - 4 = 6x$$

$$\frac{y-4}{6} = x$$

swap x, y

$$\frac{x-4}{6} = y$$

rewrite

$$\frac{x-4}{6} = f^{-1}(x)$$

8. Find the domain of $f(x) = \log(2x + 15)$.

$$2x + 15 > 0$$

$$2x > -15$$

$$x > \frac{-15}{2}$$

9. The function $P(t) = 21.109 - 5.686\ln(t+1)$ describes the revenue, in thousands of dollars, for the sale of a product t weeks after an ad campaign for the product ended, where $0 \le t \le 10$. Find $P(5)$, round to the nearest cent, and interpret the meaning of $P(5)$ in a complete sentence.

$P(5) = 21.109 - 5.686\ln(5+1)$

$= 21.109 - 5.686\ln 6$

$= 10.92106$

\$10,921.06 in Revenue, 5 weeks after an ad campaign.

10. What are all the real and complex zeros of $x^3 + 125$, given that one zero is $x = -5$?

$x^3 + 125 = (x+5)(\ f(x)\ )$

	x^3	x^2	x	c
-5	1	0	0	125
		-5	25	-125
	1	-5	25	0

$f(x) = x^2 - 5x + 25$

$x = \dfrac{-(-5) \pm \sqrt{(-5)^2 - 4(25)(1)}}{2(1)}$

$= \dfrac{5 \pm \sqrt{25 - 100}}{2}$

$= \dfrac{5}{2} \pm \dfrac{i\sqrt{75}}{2}$

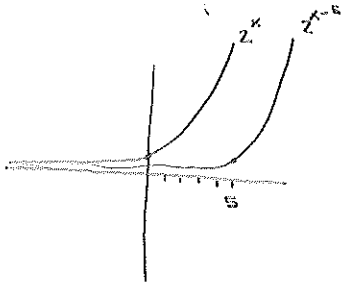
$= \dfrac{5}{2} \pm \dfrac{i5\sqrt{3}}{2}$

Real zeros: -5

Complex zeros: $\dfrac{5}{2} + \dfrac{5i\sqrt{3}}{2}$

$\dfrac{5}{2} - \dfrac{5i\sqrt{3}}{2}$

11. What is the horizontal asymptote of $f(x) = 2^{x-5}$? Explain.



The horizontal asymptote is at $y = 0$

The horizontal asymptote of the function $g(x) = 2^x$ is at $y = 0$. This function ($f(x)$) is the same as $g(x)$ but shifted to the right by 5. Therefore the asymptote did not change

12. Find a fourth degree polynomial having single roots at $x = -4$ and $x = -3$, and a double root at $x = 5$. Do not multiply your answer out.

$p(x) = (x+4)(x+3)(x-5)^2$

13. Given the revenue function $R(x) = 64x - x^3$, where x is a number of units, what numbers of units give zero revenue?

$R(x) = 64x - x^3$

$= x(64 - x^2)$

$= x(8-x)(8+x)$

$x = 0, 8, -8$

For the problem: 0 or 8 units sold gives \$0 in revenue.

14. (8 points) Given that $x = -6$ is one zero of $p(x) = x^3 + 8x^2 + 17x + 30$, find all the other zeros, real or complex, of $p(x)$.

$p(x) = (x+6)(\ f(x)\ )$

$$\begin{array}{r|cccc} -6 & 1 & 8 & 17 & 30 \\ & & -6 & -12 & -30 \\ \hline & 1 & 2 & 5 & 0 \end{array}$$

Real roots : -6

Complex : $-1+2i$, $-1-2i$

$f(x) = x^2 + 2x + 5$

$x = \dfrac{-2 \pm \sqrt{4 - 4(5)(1)}}{2(1)}$

$= \dfrac{-2 \pm \sqrt{-16}}{2}$

$= -1 \pm \dfrac{4i}{2}$

$= -1 \pm 2i$

15. Fill in the blank:

(a) $\log_5\left(\frac{1}{125}\right) =$ -3

(b) $\log_3(243) =$ 5

(c) $\log_B(\sqrt{B}) =$ $\frac{1}{2}$

16. Graph $f(x) = 2^x$ AND graph its inverse, $f^{-1}(x)$, on the same grid below. Label the graphs, include asymptotes.

$y = 2^x$

$x = 2^y$

$\log_2 x = y$

$\log_2 x = f^{-1}(x)$