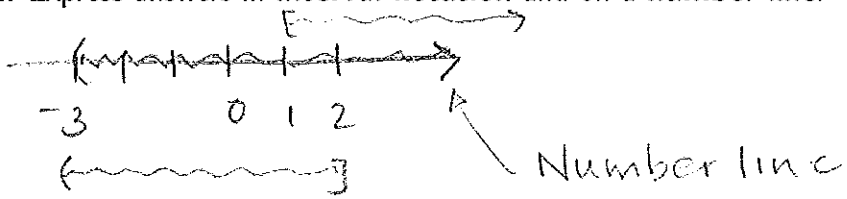
Name: Key
Recitation Instructor, Day, Time:

TRADITIONAL MATH 100 – Exam 1 – September 15, 2015

Directions: You will find 13 problems listed below. No notes/books/friends are allowed. Graphing calculator models above the level of a TI-84 plus are not allowed. You have one hour to complete this exam.

# 1	# 2	# 3	# 4	# 5	# 6	# 7	# 8	# 9	# 10	# 11	# 12	# 13	TOTAL

1. (8 points) Find the union. Express answers in **interval notation** and on a **number line**:
$(-3, 2] \cup [1, \infty)$



Interval. $(-3, \infty)$

2. (6 points) Consider the graph of $h(x) = x^2$. Describe how the graph of $h(x+4) - 5$ would look in terms of translations.

$h(x+4)-5$ has the shape of $h(x)=x^2$, but shifted left 4 units and down 5 units.

3. (8 points) Considser the two points $\overset{x_1\ y_1}{(4, -9)}$ and $\overset{x_2\ y_2}{(-6, 7)}$.

(a) Find the midpoint of these points.

$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$; $\left(\frac{4-6}{2}, \frac{-9+7}{2}\right)$; $\boxed{(-1, -1)}$

(b) Find the distance between these points.

$$\text{distance} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(-10)^2+(16)^2} = \sqrt{100+256} = \sqrt{356} = \boxed{2\sqrt{89}}$$

4. (10 points) Evaluate and complete the following function table for $f(t) = t^2 + 3Mt$, where M is some unspecified parameter. Show all work.

t	-2	-1	0	1	2
$f(t)$	$4-6M$	$1-3M$	0	$1+3M$	$4+6M$

$f(-2) = (-2)^2 + 3M(-2) = 4 - 6M.$

$f(-1) = (-1)^2 + 3M(-1) = 1 - 3M$

$f(0) = 0 + 3M(0) = 0$

$f(1) = 1^2 + 3M(1) = 1 + 3M$

$f(2) = 2^2 + 3M(2) = 4 + 6M$

5. (6 points) Solve for x: $4(x-1)+7 = 7(x-2)-5$

$4x - 4 + 7 = 7x - 14 - 5$

$4x + 3 = 7x - 19$

$-3x = -22$

$x = \frac{22}{3}$

6. (8 points) Is the function $f(x) = x^2 - x + 1$ even, odd, or neither? Use the definitions of even/odd to justify your answers.

$f(-x) = (-x)^2 - (-x) + 1 = x^2 + x + 1$

$-f(x) = -1(x^2 - x + 1) = -x^2 + x - 1$

Since $f(x) \neq f(-x)$, f isn't even

Since $-f(x) \neq f(-x)$, f isn't odd.

Therefore, the answer is NEITHER.

7. (8 points) Given $f(x) = 4x - 7$, find the difference quotient $\dfrac{f(x+h) - f(x)}{h}$.

$f(x+h) = 4(x+h) - 7 = 4x + 4h - 7$.

$$\frac{f(x+h) - f(x)}{h} = \frac{4x + 4h - 7 - (4x - 7)}{h}$$

$$= \frac{4x + 4h - 7 - 4x + 7}{h} = \frac{4h}{h} = \boxed{4}$$

8. (8 points) Suppose the cost function for a certain product is given by $C(x) = 12x + 300$ and the revenue function for the product is given by $R(x) = 40x$. Find a formula for the following functions:

(a) Profit Function, $P(x)$

$P(x) = R(x) - C(x) = 40x - (12x + 300) = 28x - 300$.

(b) Average Cost Function, $\overline{C(x)}$

$$\overline{C(x)} = \frac{C(x)}{x} = \frac{12x + 300}{x} \quad \left(\text{or } \overline{C(x)} = 12 + \frac{300}{x}\right)$$

9. (10 points) In a controlled lab environment, some organisms exhibit constant growth over a specific time period. Suppose a certain organism starts out weighing 4 mg, and grows to 16 mg over a 24 hour time period. Find a linear model that describes the growth of the organism for $0 \leq t \leq 24$ hours.

t	weight
0	4
24	16.

Slope: $\dfrac{16 - 4}{24 - 0} = \dfrac{12}{24} = \dfrac{1}{2}$.

$y - y_0 = m(t - t_0)$

$y - 4 = \frac{1}{2}(t - 0)$.

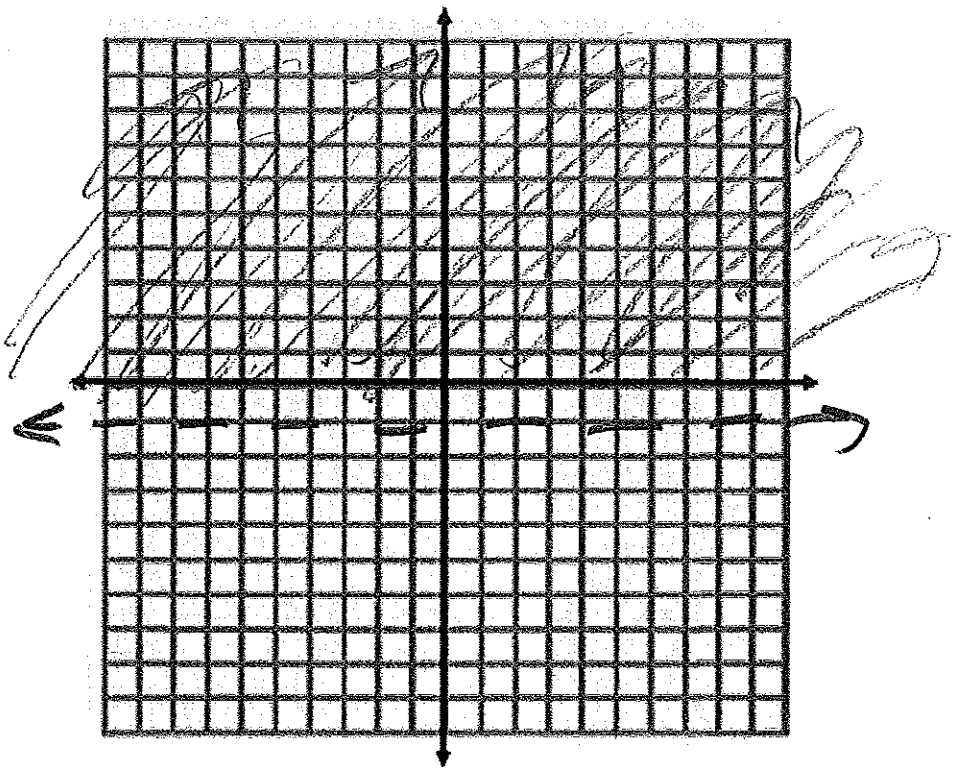
either one fine.

t: time

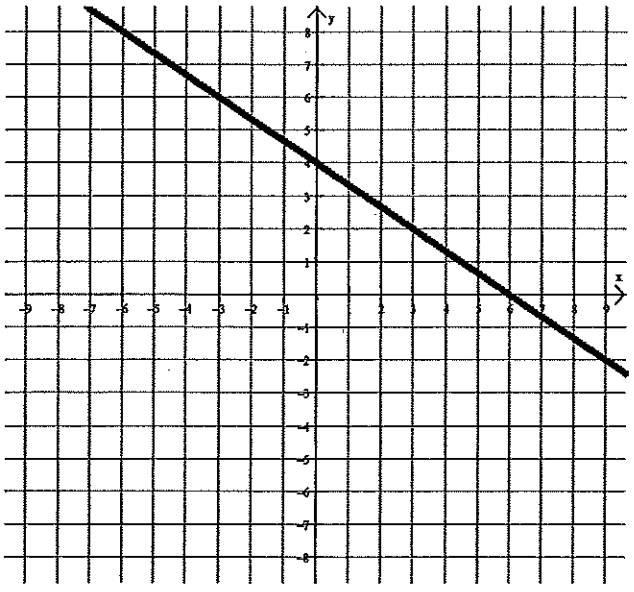
y: weight

$$\boxed{y = \frac{1}{2}t + 4}$$

10. (6 points) On the grid below, graph the relation $\{(x, y)|y > -1\}$.



11. (6 points) Find a formula for the linear function given below:



y-intercept is (0, 4).

Slope is $-\frac{4}{6} = -\frac{2}{3}$

$y = -\frac{2}{3}x + 4$

OR

$y - 4 = -\frac{2}{3}(x - 0)$

OR

$y - 0 = -\frac{2}{3}(x - 6)$

12. (8 points) Find an equation of the line passing through $(-3, 5)$ and perpendicular to $4x - 2y = 8$.

Given line in slope-intercept form:

$-2y = -4x + 8$

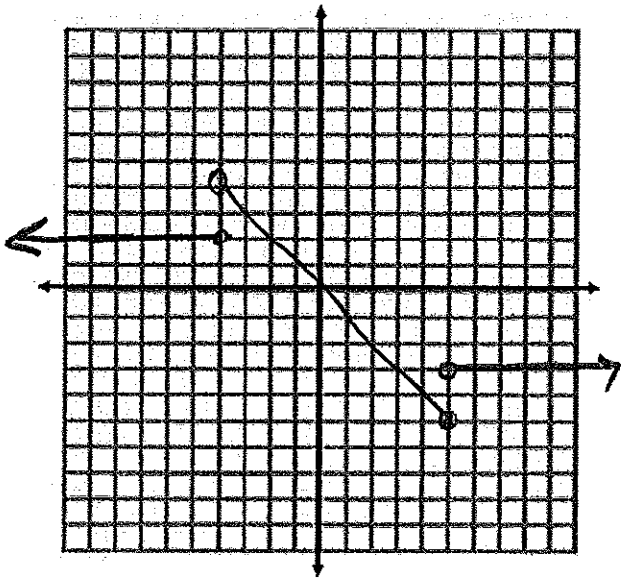
$y = 2x - 4.$

Slope of new line: $-\frac{1}{2}$.

New line: $\boxed{y - 5 = -\frac{1}{2}(x + 3)}$ OR $y = -\frac{1}{2}x + \frac{7}{2}$

13. (8 points) Graph the following piecewise function on the grid given below.

$$f(x) = \begin{cases} 2, & x \leq -4 \\ -x, & -4 < x \leq 5 \\ -3, & x > 5 \end{cases}$$



(a) Over what x-intervals is the graph of $g(x)$ increasing?

none.

(b) Over what x-intervals is the graph of $g(x)$ decreasing?

$(-4, 5)$.