Name: Key

Recitation Instructor, Day, Time:

TRADITIONAL MATH 100 – Exam 3 – November 10, 2015

Directions: You will find 12 problems listed below. No notes/books/friends are allowed. Graphing calculator models above the level of a TI-84 plus are not allowed. You have one hour to complete this exam.

# 1	# 2	# 3	# 4	# 5	# 6	# 7	# 8	# 9	# 10	# 11	# 12	TOTAL

1. (a) (6 points) Find $f^{-1}(x)$ when $f(x) = \dfrac{5x+1}{2}$.

$$y = \frac{5x+1}{2}$$

$$2y = 5x + 1$$

$$2y - 1 = 5x$$

$$x = \frac{2y-1}{5}$$

Answer: $f^{-1}(x) = \dfrac{(2x-1)}{5}$

(b) (6 points) Find $g^{-1}(x)$ when $g(x) = \log_3(2x+7)$.

$$y = \log_3(2x+7)$$

$$3^y = 2x + 7$$

$$3^y - 7 = 2x$$

$$x = \frac{(3^y - 7)}{2}$$

Answer: $g^{-1}(x) = \dfrac{(3^x - 7)}{2}$

2. (10 points) Condense into a single logarithmic expression: $\log_6(x) + \log_{36}(x+1)$. (Hint: Change of base formula).

$$\log_6(x) + \log_{36}(x+1) = \log_6(x) + \frac{\log_6(x+1)}{\log_6(36)}$$

$$= \log_6(x) + \frac{\log_6(x+1)}{2}$$

$$= \log_6(x) + \frac{1}{2}\log_6(x+1)$$

$$= \log_6(x) + \log_6(x+1)^{1/2}$$

$$= \log_6\left(x(x+1)^{1/2}\right)$$

* Converting to base 36 correctly is fine also!

3. (8 points) Using the values $\log(a) = 1.4$ and $\log(b) = 2.2$, find $\log\left(\sqrt{ab^3}\right)$.

$$\log(\sqrt{ab^3}) = \log(ab^3)^{1/2}$$
$$= \frac{1}{2}\left[\log a + 3\log b\right]$$
$$= \frac{1}{2}\left[1.4 + 3(2.2)\right]$$
$$= \frac{1}{2}\left[1.4 + 6.6\right] = \frac{1}{2}[8] = \boxed{4}$$

or $\log(\sqrt{ab^3}) = \log(ab^3)^{1/2} = \log(a^{1/2}b^{3/2})$
$$= \log(a^{1/2}) + \log(b^{3/2})$$
$$= \frac{1}{2}\log a + \frac{3}{2}\log b = \frac{1}{2}(1.4) + \frac{3}{2}(2.2)$$
$$= \boxed{4}$$

4. (8 points) Solve the following rational equation: $\dfrac{3x-4}{x-1} = \dfrac{6x}{2x-3}$

$$\frac{3x-4}{x-1} - \frac{6x}{2x-3} = 0.$$

$$\frac{(3x-4)(2x-3) - 6x(x-1)}{(x-1)(2x-3)} = 0$$

$$\frac{6x^2 - 9x - 8x + 12 - 6x^2 + 6x}{(x-1)(2x-3)} = 0.$$

$$\frac{-11x + 12}{(x-1)(2x-3)} = 0.$$

Setting numerator equal to 0 gives $-11x + 12 = 0$

So $\boxed{x = \frac{12}{11}}$

5. (8 points) Solve: $5 + \ln(x+2) = 7$. Leave answers exact (in other words, don't use a calculator).

Method 1

$5 + \ln(x+2) = 7$

$\ln(x+2) = 2$

$e^2 = x + 2$

$\boxed{e^2 - 2 = x}$

Method 2:

$5 + \ln(x+2) = 7$

$\ln(x+2) = 2$

$e^{\ln(x+2)} = e^2$

$x + 2 = e^2$

$\boxed{x = e^2 - 2}$

6. (8 points) Solve: $2 + 7e^x = 11$. Leave answers exact (in other words, don't use a calculator).

$7e^x = 9$

$e^x = \frac{9}{7}$

$\boxed{\ln\left(\frac{9}{7}\right) = x.}$

Method 2:

$\ln e^x = \ln\left(\frac{9}{7}\right)$

$x \ln e = \ln\left(\frac{9}{7}\right)$

$\boxed{x = \ln\left(\frac{9}{7}\right)}$

7. (3 points each, no partial credit) Fill in the blank:

(a) $\log_b\left(\sqrt{b}\right) =$ $\underline{1/2}$

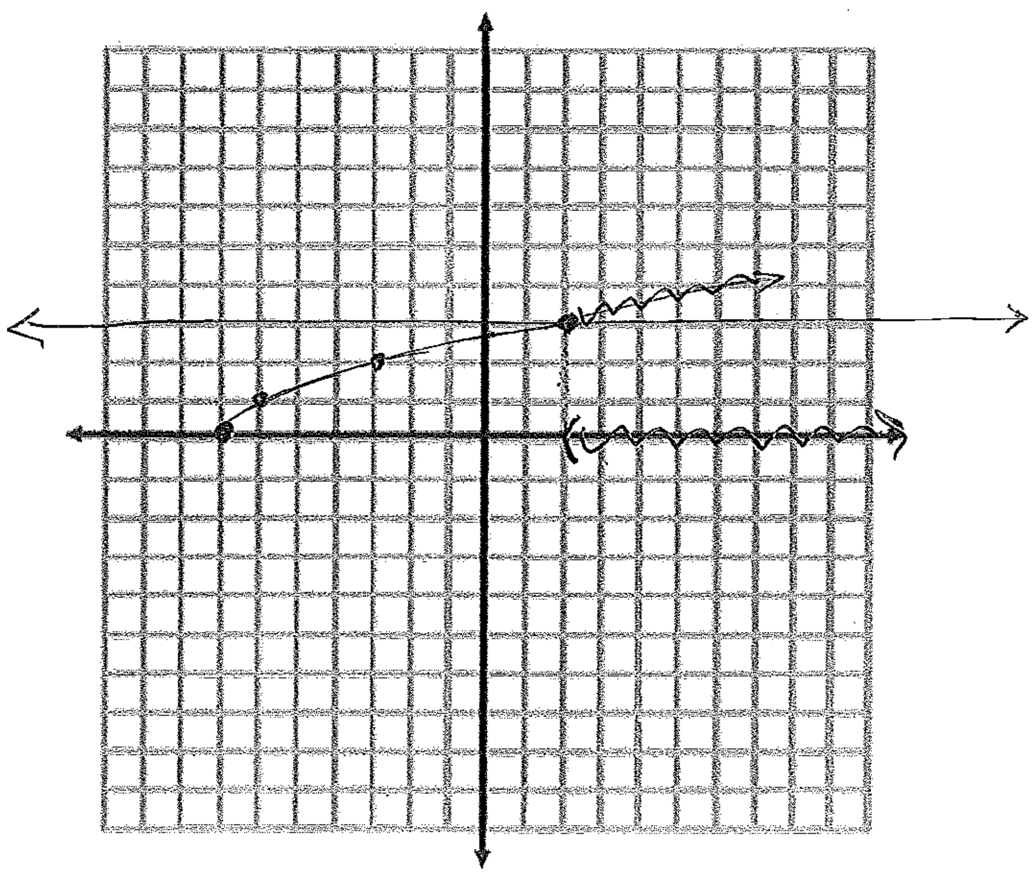
(b) $\log_3\left(\frac{1}{243}\right) =$ $\underline{-5}$

(c) $\ln(e^6) =$ $\underline{6}$

8. (8 points) Given $g(x) = x^2 - 5x - 1$ and $h(x) = -3x + 4$, find $g(h(x))$ and write your answer in the form $ax^2 + bx + c$.

$$g(h(x)) = g(-3x+4)$$
$$= (-3x+4)^2 - 5(-3x+4) - 1$$
$$= 9x^2 - 24x + 16 + 15x - 20 - 1$$
$$= \boxed{9x^2 - 9x - 5}$$

9. (8 points) Solve the inequality by graphing: $\sqrt{x+7} > 3$



Solution: $(2, \infty)$

or $x > 2$.

Method 2: (test value method)

test $x=-4$ | test $x=0$ | test $x=6$.

$\frac{-9}{-1}$ $+$ $\quad -3 \quad$ $\frac{-}{+}$ $-$ $\quad 5 \quad$ $\frac{+}{+}$ $+$.

$\boxed{\text{Solution: } -3<x<5}$

10. (8 points) Solve the rational inequality: $\dfrac{x-5}{x+3} < 0$.

Method 1:

$x-5>0$ AND $x+3<0$ or $x-5<0$ AND $x+3>0$

(*) $x>5$ AND $x<-3$ or $x<5$ AND $x>-3$.

$-3 \qquad 5$ $\qquad\qquad$ $-3 \qquad 5$

There is no x that satisfies (*), so the answer is → $\boxed{\text{Solution: } -3<x<5}$

11. (5 points) Find the domain of the function $f(x) = \log(6x+11)$.

$6x+11>0$.

$6x>-11$

$\boxed{x>-\frac{11}{6}}$.

12. (2 points each, no partial credit, even if you mix up answers between parts.) Consider the rational function $r(x) = \dfrac{16x^2+8x+1}{4x^2-1}$.

(a) What is the domain of $r(x)$?

$4x^2-1 = (2x+1)(2x-1)$; $\boxed{\text{Domain is all reals except } x=\pm\frac{1}{2}}$

(b) What are the zero(s) of $r(x)$?

$16x^2+8x+1=0$

$(4x+1)(4x+1)=0$

$\boxed{x=-1/4}$.

(c) What is the y-intercept of $r(x)$?

$r(0) = \frac{1}{-1}$. $\boxed{(0,-1)}$

(d) Does $r(x)$ have a horizontal asymptote? If so, what is it?

Yes; $y=\frac{16}{4}=4$. ('since degree of numerator/denominator are equal')