Key

Name:
Recitation Instructor:
Recitation Day and Time: 24

College Algebra – FINAL EXAM – December 2015

Page 1 16 pts.	Page 2 19 pts.	Page 3 26 pts.	Page 4 20 pts.	Page 5 27 pts.	Page 6 16 pts.	Page 7 26 pts.	TOTAL 150 pts
✓	✓	✓	✓	✓		✓	

Directions: You will find 20 problems listed below. The point value of each problem is given in parentheses. Please show all your work neatly and box your final answers. No notes or books are allowed. Graphing calculator models above the level of a TI-84 plus are not allowed (in particular, calculators with a built-in CAS and/or QWERTY keyboard are not allowed). You have one hour and fifty minutes to complete this exam.

1. (8 points) Find an equation of the line passing through the points $(2, 5)$ and $(9, 9)$.

Slope: $\frac{9-5}{9-2} = \frac{4}{7}$

Line: $y - 5 = \frac{4}{7}(x-2)$

or $y - 9 = \frac{4}{7}(x-9)$

or $y = \frac{4}{7}x + \frac{27}{7}$

other ways to get 'b':

$y = mx + b$
$5 = \frac{4}{7}(2) + b$
$\frac{27}{7} = b$

$y = mx + b$
$9 = \frac{4}{7}(9) + b$
$\frac{27}{7} = b$

2. (8 points) In a controlled lab environment, some organisms exhibit constant growth over a specific time period. Suppose a certain organism starts out weighing 2 mg, and grows to 8 mg over a 24 hour time period. Find a linear model (use function notation!) that describes the growth of the organism for $0 \leq t \leq 24$ hours.

$(0, 2)$
$(24, 8)$

Slope $= \frac{8-2}{24-0} = \frac{6\text{mg}}{24} = \frac{1}{4}$ mg/hr.

$y - 2 = \frac{1}{4}(t-0)$ or $y - 8 = \frac{1}{4}(t-24)$

$\boxed{y = \frac{1}{4}t + 2, \quad 0 \leq t \leq 24 \text{ hours}}$

3. (5 points) State TRUE or FALSE for each of the following.

(a) Linear functions are one-to-one functions. False

(b) The function $f(x) = a^x$ is an exponential growth function for any real number a with $a > 0$. False

(c) The y-intercept of a polynomial is also known as its constant term. True

(d) For a quadratic function with vertex (h, k), its axis of symmetry is $y = k$. False

(e) The range of the function $f(x) = ax^2 + bx + c$, where a, b, c are real numbers with $a \neq 0$, is all real numbers. False

4. (8 points) Solve and check: $|3x + 5| - 2 = 7$.

$3x+5 = 9$ or $3x+5 = -9$

$3x = 4$ or $3x = -14$

$x = \frac{4}{3}$ or $x = \frac{-14}{3}$

check $x = \frac{4}{3}$:

$|4+5| - 2 = 7$ ✓

check $x = -\frac{14}{3}$:

$|-14+5| - 2 = 9 - 2 = 7$ ✓

Solutions: $x = \frac{4}{3}, x = \frac{-14}{3}$

8

5. (8 points) Given $f(x) = e^x + 4$ and $g(x) = \ln(x - 4)$, find $f(g(x))$ and $g(f(x))$.

$f(g(x)) = f(\ln(x-4)) = e^{\ln(x-4)} + 4$

$= x - 4 + 4$

$= \boxed{x}$

$g(f(x)) = g(e^x + 4) = \ln(e^x + 4 - 4)$

$= \ln(e^x)$

$= x \ln e$

$= \boxed{x}$

6. (6 points) Simplify and write in standard form: $\dfrac{8-6i}{7+i}$

$$\frac{(8-6i)(7-i)}{(7+i)(7-i)} = \frac{56-8i-42i+6i^2}{49-i^2}$$

$$= \frac{50-50i}{50} = \boxed{1-i}$$

6

7. ~~(8~~ points) Solve for x the equation: $7e^{x+3} - 5 = 7$.

$$7e^{x+3} = 12$$

$$e^{x+3} = \frac{12}{7} \quad \Rightarrow \ln\left(\frac{12}{7}\right) = x+3$$

$$\boxed{\ln\left(\frac{12}{7}\right) - 3 = x}$$

$$(x \approx -2.461)$$

8. (12 points) Consider the polynomial function $p(x) = 2x^3 - 15x^2 + 22x + 15$.

(a) Using the **Rational Zero Theorem,** list all possible rational zeros of $p(x)$.

Possible rational zeros: $\dfrac{\pm 1, \pm 3, \pm 5, \pm 15}{\pm 1, \pm 2}$

$$= \left\{\pm 1, \pm\frac{1}{2}, \pm 3, \pm\frac{3}{2}, \pm 5, \pm\frac{5}{2}, \pm 15, \pm\frac{15}{2}\right\}$$

(b) Confirm that $x = 5$ is one zero of $p(x)$, and use that fact to find the other zeros of $p(x)$.

5	2	-15	22	15
	↓	10	-25	-15
	2	-5	-3	0

$2x^2 - 5x - 3 = 0$

$(2x+1)(x-3) = 0$

$\boxed{x = -\frac{1}{2}, \; x = 3}$

(c) Factor the polynomial $p(x)$ completely. In other words, write $p(x)$ as a product of linear factors.

$$p(x) = (2x+1)(x-3)(x-5)$$

9. (8 points) Solve the following radical equation, remembering to *check* your answers:
$3x - 2 = \sqrt{14x + 7}$.

$(3x-2)^2 = 14x+7$

$9x^2 - 12x + 4 = 14x + 7$

$9x^2 - 26x - 3 = 0$

$(9x+1)(x-3) = 0$

$x = -\frac{1}{9}$ or $x = 3$

Check $x = -\frac{1}{9}$:

Left side: $-\frac{1}{3} - \frac{6}{3} = -\frac{7}{3}$ ✗

Right side: $\sqrt{-\frac{14}{9}+7} = \sqrt{\frac{49}{9}} = \frac{7}{3}$

Check $x = 3$:

Left: $3(3)-2 = 7$

Right: $\sqrt{42+7} = 7$ ✓

Only $x = 3$ works.

10. (4 points) Find an equation for the graph that has the shape of $|x|$, but shifted left 5 units and downwards 2 units.

$y = |x+5| - 2$

11. (8 points)

(a) Find the zeros of the quadratic function $f(x) = -x^2 + 2x + 8$.

(b) Find the vertex of the quadratic function $f(x) = -x^2 + 2x + 8$.

(c) Find the y-intercept of $f(x)$. $(0, 8)$

(d) Using the above information, sketch a graph of $f(x)$.

$-1(x^2 - 2x - 8) = 0$

$-1(x-4)(x+2) = 0$

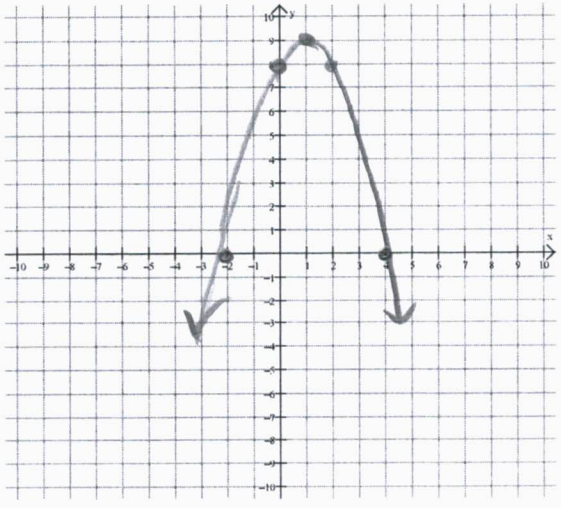
$x = 4, \; x = -2$

$h = \frac{-2}{2(-1)} = 1,$

$k = f(1)$

$= -1+2+8 = 9$

Vertex: $(1, 9)$



Note: Some may average the zeros to find the x-coordinate of the vertex.

12. (12 points) Consider the rational function $r(x) = \dfrac{(5x+2)(x+7)}{x^2-7x+12}$.

(a) Find the vertical asymptotes of $r(x)$.

$(x-3)(x-4)$; $x=3$, $x=4$

(b) Find the horizontal asymptote of $r(x)$.

$y=5$

(c) Find the zeros of $r(x)$.

$x=-\frac{2}{5}$, $x=-7$

(d) Find the y-intercept of $r(x)$.

$r(0) = \frac{14}{12} = \frac{7}{6}$ $\quad (0, \frac{7}{6})$

13. (8 points) Solve the following rational equation: $\dfrac{2x+14}{4x+7} = \dfrac{x+8}{2x+4}$.

$$\frac{(2x+14)(2x+4)-(x+8)(4x+7)}{(4x+7)(2x+4)} = 0$$

$$\frac{4x^2+8x+28x+56-[4x^2+39x+56]}{(4x+7)(2x+4)} = 0$$

$-3x = 0 \rightarrow x = 0$

14. (7 points) Suppose \$400 is invested in an account paying 5% annual interest and being compounded monthly. How much time is required for the initial investment to triple?

$P(t) = P_0\left(1+\frac{r}{n}\right)^{nt}$

$1200 = 400\left(1+\frac{.05}{12}\right)^{12t}$

$3 = \left(1+\frac{.05}{12}\right)^{12t}$

$\ln(3) = 12t\ln\left(1+\frac{.05}{12}\right)$

$t = \dfrac{\ln(3)}{12\ln\left(1+\frac{.05}{12}\right)}$

$t \approx 22.02$ years

15. (8 points) Using properties of logarithms and assuming all variables positive:

(a) Expand completely: $\log(xy^3z^2)$

$\log x + 3\log y + 2\log z$

(b) Solve for x: $3\log(x-5) = 9$.

$\log(x-5) = 3$

$10^3 = x-5$

$1005 = x$

16. (8 points) Light roast coffee beans cost \$7.00/lb, while dark roast ones cost \$5.50/lb. How much of each type of coffee is needed to create 4.50 pounds of a mixture that costs \$6.75 per pound?

Let x = amount of light roast coffee

y = amount of dark roast coffee

$$\begin{cases} x+y = 4.50 \\ 7x+5.5y = (6.75)(4.5) \end{cases}$$

$$\begin{cases} (x+y = 4.50)\cdot -7 \\ 7x+5.5y = 30.375 \end{cases}$$

$$\begin{cases} -7x-7y = -31.5 \\ 7x+5.5y = 30.375 \end{cases}$$

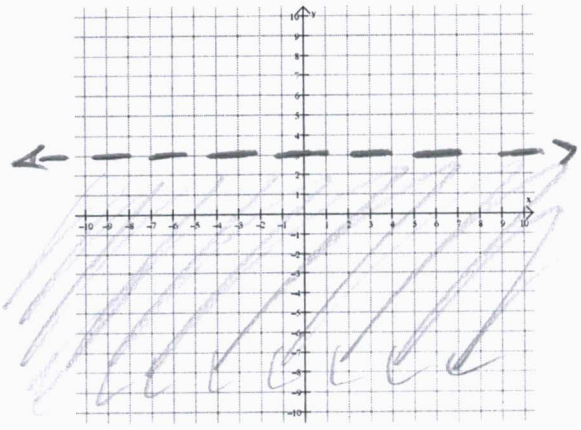
$-1.5y = -1.125$

$y = .75$ $\quad x = 3.75$

3.75 lbs of light roast

.75 lbs of dark roast

17. (6 points) On the grid below, graph the relation $\{(x, y)|y < 3\}$.



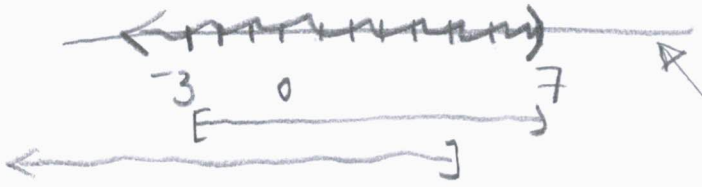
18. (8 points) Find the inverse of the following matrix:

$$\begin{pmatrix} 1 & 4 \\ 2 & -3 \end{pmatrix}$$

$$\frac{1}{-3-8}\begin{pmatrix} -3 & -4 \\ -2 & 1 \end{pmatrix} = -\frac{1}{11}\begin{pmatrix} -3 & -4 \\ -2 & 1 \end{pmatrix}$$

$$= \begin{bmatrix} 3/11 & 4/11 \\ 2/11 & -1/11 \end{bmatrix}$$

19. (6 points) Find the union. Express answers in **interval notation** and on a **number line**:
$(-\infty, 5] \cup [-3, 7)$

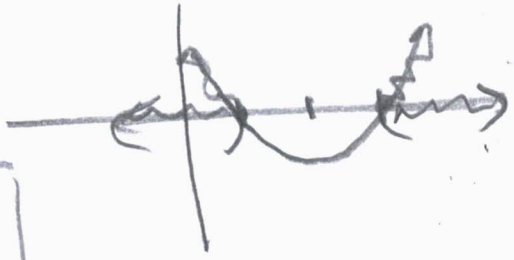


$(-\infty, 7)$

line answer

20. (6 points) Solve $x^2 - 4x + 3 > 0$.

$(x-3)(x-1) > 0$



$x > 3$ or $x < 1$

(a # line or case analysis is fine too)