Name: key
Recitation Instructor, Day, Time:

TRADITIONAL MATH 100 – Exam 2 – March 2015

Directions: You will find 14 problems listed below. No notes/books/friends are allowed. Graphing calculator models above the level of a TI-84 plus are not allowed. You have one hour to complete this exam.

Page 1 20 pts.	Page 2 20 pts.	Page 3 20 pts.	Page 4 20 pts.	Page 5 20 pts	TOTAL 100 pts
✓	✓	✓	✓	✓	

1. (6 points) Find the solutions and check your answers: $10 - 2|x + 1| = 4$.

$10 - 2|x+1| = 4$
$-2|x+1| = -6$
$|x+1| = 3$
$x+1 = 3$ or $x+1 = -3$
$x = 2$ or $x = -4$

check $x = 2$: $10 - 2|2+1|$
$= 10 - 2|3|$
$= 10 - 2(3) = 4$ ✓

check $x = -4$: $10 - 2|-4+1|$
$= 10 - 2|-3|$
$= 10 - 2(3) = 4$ ✓

2. (6 points) Find the solutions to $2x^2 + x - 3 = 0$.

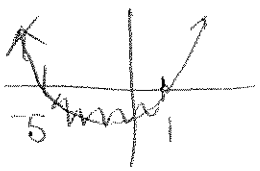
$a = 2, b = 1, c = -3$

$x = \frac{-1 \pm \sqrt{1 - 4(2)(-3)}}{2(2)} = \frac{-1 \pm \sqrt{1+24}}{4} = \frac{-1 \pm 5}{4} = \{$

$x = -\frac{3}{2}$ or $x = 1$

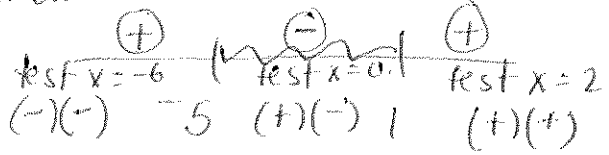
3. (8 points) Solve the quadratic inequality $x^2 + 4x < 5$.

Graphically: $x^2 + 4x - 5 < 0$
$(x+5)(x-1) < 0$



$-5 < x < 1$

OR Number Line:



4. (8 points) In a controlled lab environment, some organisms exhibit constant growth over a specific time period. Suppose a certain organism starts out weighing 2 mg, and grows to 10 mg over a 6 hour time period. Find a linear model that describes the growth of the organism for $0 \leq t \leq 6$ hours.

$(0, 2\text{mg})$ ← y-intercept.
$(6, 10\text{mg})$

Slope $= \frac{10-2}{6-0} = \frac{8}{6} = \frac{4}{3}$.

$\boxed{y = \frac{4}{3}t + 2}$ OR $y - 2 = \frac{4}{3}(t)$ OR $y - 10 = \frac{4}{3}(t - 6)$

5. (6 points) Find an equation of the line passing through $(1, 5)$ and parallel to $x - 2y = 2$.

$2y = x - 2$
$y = \frac{1}{2}x - 1$.

new line: $y - 5 = \frac{1}{2}(x - 1)$

or $y = \frac{1}{2}x + \frac{9}{2}$.

6. (6 points) Find the quotient and remainder when $p(x) = 2x^3 - 7x + 3$ is divided by $x^2 + 2x - 1$. Write $p(x)$ in the form $d(x)q(x) + r(x)$, where $d(x), q(x)$ and $r(x)$ are the divisor, quotient and remainder, respectively.

$$\begin{array}{r} 2x - 4 \\ x^2+2x-1 \overline{)\,2x^3+0x^2-7x+3} \\ -(2x^3+4x^2-2x) \\ \hline -4x^2-5x+3 \\ -(-4x^2-8x+4) \\ \hline 3x-1 \end{array}$$

$\boxed{p(x) = (x^2+2x-1)(2x-4) + (3x-1)}$.

7. (10 points) The cost function for selling x units of a certain product is given by $C(x) = x^2 + 200x + 42000$.

(a) What is the vertex of this quadratic function? Show your work with algebra. If you choose to use a graph as part of your work, you must include a graph having the pertinent information that helps to answer this question.

$(h, k) = \left(-\frac{b}{2a}, C(h)\right)$. vertex: $(100, 32000)$

$h = \frac{200}{2(1)} = 100$

$k = C(100)$
$= 100^2 + 200(100) + 42000$
$= 32000.$

(b) Interpret the meaning of the vertex in context of the situation.

Minimum cost of $32000 occurs when 100 units are made.

8. (10 points) Consider the polynomial $p(x) = 3x^5 - 7x^2 + 2x + 200$. Circle TRUE or FALSE for each of the statements below.

(a) TRUE (FALSE) $p(x)$ has even degree.

(b) TRUE (FALSE) $p(x)$ has a negative y-intercept.

(c) (TRUE) FALSE $p(x)$ has positive leading coefficient.

(d) (TRUE) FALSE As $x \to \infty$, $p(x) \to \infty$.

(e) TRUE (FALSE) As $x \to -\infty$, $p(x) \to \infty$.

9. (8 points) A parabola has vertex at $(1,5)$ and passes through the point $(-2,9)$. What is the equation of the parabola? Write your answer in the form $y = ax^2 + bx + c$.

$y = a(x-h)^2 + k$

$9 = a(-2-1)^2 + 5$

$9 = a(9) + 5$

$4 = 9a$

$a = \frac{4}{9}$

$y = \frac{4}{9}(x-1)^2 + 5$

$y = \frac{4}{9}(x^2 - 2x + 1) + 5$

$y = \frac{4}{9}x^2 - \frac{8}{9}x + \frac{4}{9} + \frac{45}{9}$

$\boxed{y = \frac{4}{9}x^2 - \frac{8}{9}x + \frac{49}{9}}$

10. (6 points) Using the **REMAINDER THEOREM**, find $p(-3)$ when $p(x) = 2x^4 - 4x^2 + x - 1$. Be sure to identify your final answer.

-3	2	0	-4	1	-1
	$\downarrow$	-6	18	-42	123
	2	-6	14	-41	$\boxed{122}$

$\boxed{p(-3) = 122}$

11. (6 points) Two parabolas have functions given by $f(x) = -x^2 + 2x$ and $g(x) = x^2 + 7x + 2$. Find the intersection points of the two parabolas and state your answers as ordered pairs.

$-x^2 + 2x = x^2 + 7x + 2$

$0 = 2x^2 + 5x + 2$

$0 = (2x+1)(x+2)$

$x = -\frac{1}{2} \rightsquigarrow y = -\frac{1}{4} + 2(-\frac{1}{2}) = -\frac{5}{4}$

$x = -2 \rightsquigarrow y = -4 - 4 = -8$

$\boxed{(-\frac{1}{2}, -\frac{5}{4}) \text{ and } (-2, -8)}$

12. (6 points) Solve: $|4x - 2| < 10$.

$-10 < 4x - 2 < 10$.

$-10 < 4x - 2$ and $4x - 2 < 10$

$-8 < 4x$ and $4x < 12$

$-2 < x$ and $x < 3$

$\boxed{-2 < x < 3}$

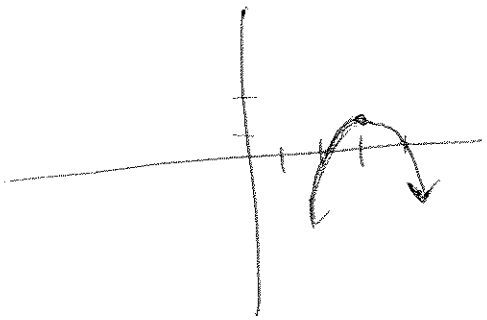
13. (6 points) Solve: $|2x - 5| > 8$.

$2x - 5 > 8$ or $2x - 5 < -8$

$2x > 13$ or $2x < -3$.

$\boxed{x > \frac{13}{2} \text{ or } x < -\frac{3}{2}}$

14. (8 points) Consider the parabola $f(x) = -(x-3)^2 + 1$. Answer the following questions. (Drawing a quick sketch of the graph of $f(x)$ may help you.)



(a) What is the domain of $f(x)$?

all Reals.

(b) What is the vertex of $f(x)$?

$(3, 1)$

(c) What is the range of $f(x)$?

$(-\infty, 1]$

(d) What is the axis of symmetry of $f(x)$?

$x = 3$