Carmè

Name: Key
Recitation Instructor, Day, Time:

TRADITIONAL MATH 100 – Exam 2 – October 2016

Directions: You will find 16 problems listed below. SHOW ALL WORK! No notes/books/friends are allowed. Graphing calculator models above the level of a TI-84 plus are not allowed. You have one hour to complete this exam.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	TOTAL

1. (6 points) Find the solutions and check your answers: $13 + |x + 7| = 25$.

$|x+7| = 12$

$x+7 = 12$ or $x+7 = -12$

$x = 5$ or $x = -19$

Solutions: $x = 5$ or $x = -19$

Check $x=5$: $13 + |5+7| = 13+12 = 25$ ✓

$x = -19$: $13 + |-19+7| = 13+12 = 25$ ✓

2. (6 points) Find the solutions to $x^2 + 10x - 2 = 0$.

$a = 1$, $b = 10$, $c = -2$

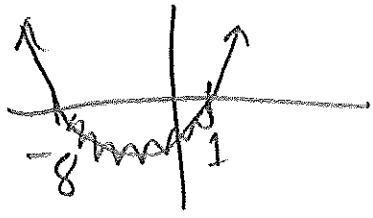
$$x = \frac{-10 \pm \sqrt{100 - 4(1)(-2)}}{2(1)}$$

$$x = \frac{-10 \pm \sqrt{108}}{2} = \frac{-10}{2} \pm \frac{6\sqrt{3}}{2} = \boxed{-5 \pm 3\sqrt{3}}$$

3. (6 points) Solve the quadratic inequality $x^2 + 7x < 8$.

Graph method: $x^2 + 7x - 8 < 0$

$(x+8)(x-1) < 0$



$\boxed{-8 < x < 1}$

(# line method also fine)

Case Analysis method:

$\{x+8 > 0$ AND $x-1 < 0\}$

or

$\{x+8 < 0$ AND $x-1 > 0\}$;

$x > -8$ AND $x < 1$

or

$x < -8$ AND $x > 1$;

So $-8 < x < 1$ only works

Bret

4. (6 points) Given that $x = 6$ is one zero of $p(x) = x^3 - 216$, find all the other zeros, real or complex, of $p(x)$.

$$\begin{array}{r|rrr} 6 & 1 & 0 & -216 \\ & \downarrow & 6 & 36 & 216 \\ \hline & 1 & 6 & 36 & 0 \end{array}$$

$x^2 + 6x + 36 = 0$;

$$x = \frac{-6 \pm \sqrt{36 - 4(1)(36)}}{2(1)} \; ; \quad x = \frac{-6 \pm 6\sqrt{-3}}{2} = \boxed{-3 \pm 3\sqrt{3}\, i}$$

5. (6 points) Suppose a rational function has poles at $x = -7$ and $x = 1$, zeros at $x = 4$ and $x = -2$, and a horizontal asymptote $y = 5$. Find a possible rational function that has such attributes.

$$r(x) = \frac{5(x-4)(x+2)}{(x+7)(x-1)}$$

Jiang

6. (6 points) Find the quotient and remainder when $p(x) = 4x^3 - 3x^2 + 1$ is divided by $x^2 + x + 5$. Write $p(x)$ in the form $d(x)q(x) + r(x)$, where $d(x), q(x)$ and $r(x)$ are the divisor, quotient and remainder, respectively.

$$\begin{array}{r} 4x - 7 \\ x^2 + x + 5 \;\overline{)\; 4x^3 - 3x^2 + 0x + 1} \\ -(4x^3 + 4x^2 + 20x) \\ \hline -7x^2 - 20x + 1 \\ -(-7x^2 - 7x - 35) \\ \hline -13x + 36 \end{array}$$

$$p(x) = (x^2 + x + 5)(4x - 7) + (-13x + 36)$$

CJ

7. (6 points) The profit function for selling x units of a certain product is given by $P(x) = -x^2 + 600x + 4800$. What number of units generates maximum profit, and, what is the maximum profit? Show your work with algebra. If you choose to use a graph as part of your work, you must include a graph having the pertinent information that helps to answer this question.

vertex is a maximum;
Find (h, k), the vertex.

$h = \frac{-b}{2a}$; $h = \frac{-600}{2(-1)} = 300$ units

$P(300) = -300^2 + 600(300) + 4800$

$= -90000 + 180000 + 4800$

$= 90000 + 4800 = \$94800$ profit

8. (6 points) Simplify and write in standard $a + bi$ form: $(-6 + 3i)(9 - 4i)$

$(-6+3i)(9-4i)$

$= -54 + 24i + 27i - 12i^2$

$= -54 + 51i + 12 = \boxed{-42 + 51i}$

9. (8 points) Consider the polynomial $p(x) = (3x - 2)(x + 1)(2 - x)(x + 5)$. Circle TRUE or FALSE for each of the statements below.

(a) TRUE FALSE $p(x)$ has even degree.

(b) TRUE FALSE $p(x)$ has a negative y-intercept.

(c) TRUE FALSE $p(x)$ has positive leading coefficient.

(d) TRUE FALSE As $x \to \infty$, $p(x) \to \infty$.

Prem

10. (6 points) A parabola has vertex at $(1, 7)$ and passes through the point $(3, 4)$. What is the equation of the parabola? Write your answer in the form $y = ax^2 + bx + c$.

$y = a(x-h)^2 + k$

$4 = a(3-1)^2 + 7$

$4 = 4a + 7$

$-3 = 4a$

$-\frac{3}{4} = a$

$y = -\frac{3}{4}[(x-1)(x-1)] + 7$

$y = -\frac{3}{4}[x^2 - 2x + 1] + 7$

$\boxed{y = -\frac{3}{4}x^2 + \frac{3}{2}x + \frac{25}{4}.}$

11. (4 points) Using the **REMAINDER THEOREM**, find $p(-1)$ when $p(x) 5x^4 - 3x^2 + x + 7$. Be sure to identify your final answer.

$p(x) =$

-1	5	0	-3	1	7
	↓	-5	5	-2	1
	5	-5	2	-1	8

$\boxed{p(-1) = 8}$

12. (6 points) Simplify i^{2441}.

$i^{2441} = i^{4(610)+1} = (i^4)^{610} \cdot i^1$

$= (1)(i)$

$= \boxed{i}$.

Lauren

13. (6 points) Solve: $|2x - 1| < 9$.

$-9 < 2x - 1$ and $2x - 1 < 9$

$-8 < 2x$ and $2x < 10$

$-4 < x$ and $x < 5$

$\boxed{-4 < x < 5}$

14. (6 points) Solve: $|2x + 5| > 3$.

$2x + 5 > 3$ or $2x + 5 < -3$

$2x > -2$ or $2x < -8$

$\boxed{x > -1 \quad \text{or} \quad x < -4}$

Horace

15. (8 points) Consider the parabola $f(x) = (x-4)^2 - 7$. Answer the following questions. (Drawing a quick sketch of the graph of $f(x)$ may help you.)

(a) What is the domain of $f(x)$?

All real numbers.

(b) What is the vertex of $f(x)$?

$(4, -7)$

(c) What is the range of $f(x)$?

$[-7, \infty)$

(d) What is the axis of symmetry of $f(x)$?

$x = 4$

16. (6 points)Find all the zeros of $p(x) = x^4 - 13x^2 + 36$.

$(x^2-9)(x^2-4) = 0$

$(x-3)(x+3)(x-2)(x+2) = 0$

$\boxed{x = \pm 3, \; x = \pm 2}$