Name: Key
Recitation Instructor, Day, Time:

TRADITIONAL MATH 100 – Exam 3 – November 2016

Directions: You will find 13 problems listed below. No notes/books/friends are allowed. Graphing calculator models above the level of a TI-84 plus are not allowed. You have one hour to complete this exam.

# 1	# 2	# 3	# 4	# 5	# 6	# 7	# 8	# 9	# 10	# 11	# 12	# 13	TOTAL

1. (a) (6 points) Find $f^{-1}(x)$ when $f(x) = 5x + 1$.

$$y = 5x + 1$$
$$y - 1 = 5x$$
$$\frac{y-1}{5} = x$$

$$\boxed{f^{-1}(x) = \frac{x-1}{5}}$$

(b) (6 points) Find $g^{-1}(x)$ when $g(x) = \log_4(7x + 3)$.

$$y = \log_4(7x+3)$$
$$4^y = 7x + 3$$
$$4^y - 3 = 7x$$
$$\frac{4^y - 3}{7} = x$$

$$\boxed{f^{-1}(x) = \frac{4^x - 3}{7}}$$

2. ($\cancel{7}$ 8 points) Given $g(x) = x^2 + 2x - 1$ and $h(x) = 3x + 4$, find $g(h(x))$ and write your answer in the form $ax^2 + bx + c$.

$$g(3x+4) = (3x+4)^2 + 2(3x+4) - 1$$
$$= 9x^2 + 24x + 16 + 6x + 8 - 1$$
$$= \underline{9x^2 + 30x + 23}$$

3. (6 points) Using the values $\log(a) = 1.6$ and $\log(b) = 2.4$, find $\log\left(\sqrt{a^3 b}\right)$.

$$\log(a^3b)^{1/2} = \frac{1}{2}\left[\log(a^3b)\right]$$

$$= \frac{1}{2}\left[3\log a + \log b\right]$$

$$= \frac{1}{2}\left[3(1.6) + 2.4\right]$$

$$= 2.4 + 1.2$$

$$= \boxed{3.6}$$

4. (6 points) Solve: $2 + \ln(x-1) = 9$. Leave answers exact (in other words, don't use a calculator).

$$\ln(x-1) = 7$$

$$e^7 = x - 1$$

$$\boxed{e^7 + 1 = x}$$

5 (8 points) Condense into a single logarithmic expression using the properties of logarithms (you may assume that x is positive): $\log_4(x) + \log_{16}(x+5)$. (Hint: Change of base formula).

$$\log_4(x) + \frac{\log_4(x+5)}{\log_4(16)}$$

* one could use base 16 or another base...

$$= \log_4(x) + \frac{1}{2}\log_4(x+5)$$

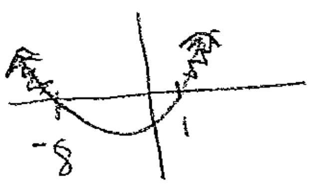
$$= \log_4(x) + \log_4(x+5)^{1/2} = \boxed{\log_4\left(x\cdot(x+5)^{1/2}\right)}$$

6 (8 points) Find the domain of the function $f(x) = \sqrt{x^2 + 7x - 8}$

$$x^2 + 7x - 8 \geq 0$$

$$(x+8)(x-1) \geq 0$$

$$\boxed{\text{Domain: } x \leq -8 \text{ or } x \geq 1}$$



7. (8 points) Solve the rational inequality: $\dfrac{x-6}{x+1} < 0$. Be sure to include either a case analysis, or, a number line justifying how you arrived at the answer.

+		−		+
test $x=-2$	-1	test $x=0$	6	test $x=7$
$\dfrac{-}{-}$		$\dfrac{-}{+}$		$\dfrac{+}{+}$

$\boxed{\text{Solution. } -1 < x < 6}$

8. (4 points each, no partial credit) Fill in the blank:

(a) $\log_b\left(\sqrt{b}\right) =$ $\underline{\ 1/2\ }$

(b) $\log_5\left(\dfrac{1}{125}\right) =$ $\underline{\ -3\ }$

(c) $\ln(e^3) =$ $\underline{\ 3\ }$

$40(3^q)$

9. (6 points) The supply function for a certain product is given by $p = \cancel{300(2^q)}$, where p is the price of the product and q is the quantity supplied at that price. If the price of the product is ~~\$2400~~, how many units will be supplied?

\$3240

$3240 = 40(3^q)$

$81 = 3^q$

$\boxed{q = 4 \text{ units}}$

10. (6 points) Find the domain of the function $f(x) = \log(3x - 2)$.

$3x - 2 > 0$

$3x > 2$

$\boxed{x > \frac{2}{3}}$

12

(6 points)

11. ~~5 points~~ Suppose $R(t) = 3t + 2$ is a function that gives the radius of a circular oil spill at t minutes. Suppose $A(r)$ is the formula for area of a circle with radius r. Find an expression for $A(R(t))$.

$A(r) = \pi r^2$

leave answer in terms of pi.

$$A(r) = \pi r^2$$

$$A(R(t)) = \pi(3t+2)^2$$

$$= \pi\left[9t^2 + 12t + 4\right]$$

$$\text{or } 9\pi t^2 + 12\pi t + 4\pi$$

12. (6 points) Solve: $4 + 9e^x = 10$. Leave answers exact (in other words, don't use a calculator).

$$9e^x = 6.$$

$$e^x = \frac{2}{3}$$

$$\boxed{\ln\left(\frac{2}{3}\right) = x}$$

13. (8 points) Solve the following rational equation: $\dfrac{x+3}{x+57} = \dfrac{x+6}{5x-1}$

$$\frac{(x+3)(5x-1) - (x+6)(x+57)}{(x+57)(5x-1)} = 0$$

$$\frac{5x^2 - x + 15x - 3 - (x^2 + 63x + 342)}{(x+57)(5x-1)} = 0$$

$$\frac{4x^2 - 49x - 345}{(x+57)(5x-1)} = 0 \quad ; \quad \frac{(4x-69)(x+5)}{(x+57)(5x-1)} = 0$$

OR one may use quadratic formula

$$\boxed{x = \frac{69}{4}, \quad x = -5}$$