Bret

Name: Key
Recitation Instructor:
Recitation Day and Time:

College Algebra – FINAL EXAM – December 2016

Page 1 16 pts.	Page 2 21 pts.	Page 3 24 pts.	Page 4 20 pts.	Page 5 27 pts.	Page 6 16 pts.	Page 7 26 pts.	TOTAL 150 pts
20	19	20	22		24	18	

Directions: You will find 21 problems listed below. The point value of each problem is given in parentheses. Please show all your work neatly and box your final answers. No notes or books are allowed. Graphing calculator models above the level of a TI-84 plus are not allowed (in particular, calculators with a built-in CAS and/or QWERTY keyboard are not allowed). You have one hour and fifty minutes to complete this exam.

6 pts

1. (4 points) Solve for x: $4(x-7)+5=3(x+2)-1$

$$4x-28+5=3x+6-1$$
$$4x-23=3x+5$$
$$\boxed{x=28}$$

8 pts

2. (6 points) A line L passes through the points $(2,5)$ and $(4,b)$ where b is some real number.

   (a) For what value of b does the line L have zero slope?

$$\frac{b-5}{4-2}=0 \;;\quad b-5=0 \;;\quad \boxed{b=5}$$

   (b) For what value of b does the line L have a slope of 3?

$$\frac{b-5}{4-2}=3 \;;\quad b-5=6 \;;\quad \boxed{b=11}$$

3. (6 points) In a controlled lab environment, some organisms exhibit constant growth over a specific time period. Suppose a certain organism starts out weighing 1 mg, and grows to 9 mg over a 24 hour time period. Find a linear model (use function notation!) that describes the growth of the organism for $0 \le t \le 24$ hours.

(0, 1 mg)
(24, 9 mg)

Slope: $\frac{9-1}{24-0}=\frac{8}{24}=\frac{1}{3}$

$$\boxed{f(t)=\frac{1}{3}t+1}$$

Lauren.

4. (5 points) State TRUE or FALSE for each of the following.

   (a) Even degree polynomials are one-to-one functions. False

   (b) The function $f(x) = a^x$ is an exponential growth function for a real number a with $a > 1$. True

   (c) The y-intercept of a polynomial is also known as its constant term. True

   (d) For a quadratic function with vertex (h, k), its axis of symmetry is $x = h$. True

   (e) The range of the function $f(x) = ax^2 + bx + c$, where a, b, c are real numbers with $a \neq 0$, is all positive real numbers. False

(8 pts)

5. (8 points) Solve and check: $|3x + 5| - 2 = 14$.

$3x+5 = 16$ or $3x+5 = -16$ Both work.

$3x = 11$ or $3x = -21$

$x = \frac{11}{3}$ or $x = -7$

Check: $|3(-7)+5|-2 = |-21+5|-2$

$= |16-2$

$= 14$ ✓

Check:

$|3(\frac{11}{3})+5|-2 = |16|-2 = 14$ ✓

6. (6 points) Given $f(x) = 4x + 3$ and $g(x) = x^2 - 5$, find $f(g(x))$ and $g(f(x))$.

$f(g(x)) = f(x^2-5) = 4(x^2-5)+3$

$= 4x^2-20+3$

$= \boxed{4x^2-17}$

$g(f(x)) = g(4x+3) = (4x+3)^2-5$

$= 16x^2+24x+9-5$

$= \boxed{16x^2+24x+4}$

CJ

7. (6 points) Simplify and write in standard form: $\dfrac{8+5i}{3-i}$

$$\frac{(8+5i)(3+i)}{(3-i)(3+i)} = \frac{24+8i+15i+5i^2}{9-i^2} = \frac{19+23i}{10}$$

$$= \frac{19}{10} + \frac{23}{10}i$$

8. (6 points) Solve for x the equation: $6e^{x+1} - 2 = 10$. (Leave answer in exact form, i.e, don't use a calculator)

$$6e^{x+1} = 12$$

$$e^{x+1} = 2$$

$$\ln(2) = x+1$$

$$\boxed{\ln(2) - 1 = x}$$

9. (8 points) Consider the polynomial function $p(x) = x^4 - 3x^2 - 54$.

(a) Using the **Rational Zero Theorem,** list all possible rational zeros of $p(x)$.

$$\frac{\text{factors of constant}}{\text{factors of lead. coeff}} : \quad \left\{\frac{\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18, \pm 27, \pm 54}{\pm 1}\right\}$$

$$= \{\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18, \pm 27, \pm 54\}$$

(b) Confirm that $x = 3$ and $x = -3$ are zeros of $p(x)$, and use these facts to find the other zeros of $p(x)$.

3	1	0	-3	0	-54
	↓	3	9	18	54
	1	3	6	18	0
-3	↓	-3	0	-18	
	1	0	6	0	

$x^2 + 6 = 0$; $x^2 = -6$; $\boxed{x = \pm\sqrt{6}\,i}$ other zeros.

Horace

10. (8 points) Solve and check: $8 - x = \sqrt{x+4}$

$64 - 16x + x^2 = x + 4$

$x^2 - 17x + 60 = 0$

$(x-12)(x-5) = 0$.

$x = 12$ or $x = 5$

$x = 12$ doesn't work

$x = 5$:

$3 = \sqrt{5+4} = 9$ ✓

Only $x = 5$

6pts

11. (4 points) Find an equation for the graph that has the shape of x^3, but shifted right 4 units and downwards 1 units.

$y = (x-4)^3 - 1$

12. (8 points)

(a) Find the zeros of the quadratic function $f(x) = x^2 - 2x - 8$.

$0 = (x-4)(x+2)$

$x = 4, \; x = -2$

(b) Find the vertex of the quadratic function $f(x) = x^2 - 2x - 8$.

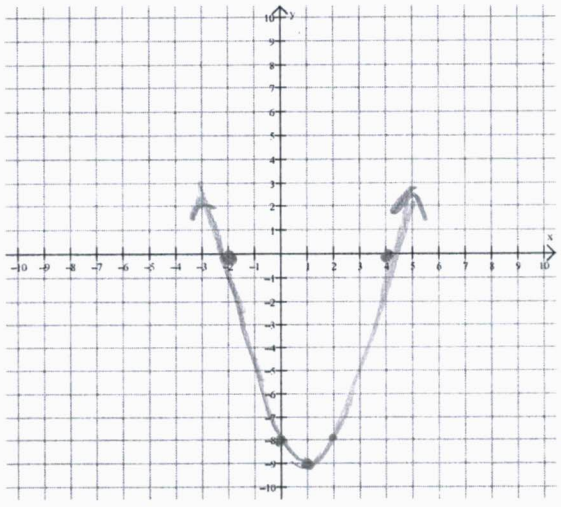
$h = \frac{2}{2(1)} = 1$; $f(1) = -9$

$(1, -9)$

(h, k)

(c) Find the y-intercept of $f(x)$. $(0, -8)$

(d) Using the above information, sketch a graph of $f(x)$.



Prem

13. (12 points) Consider the rational function $r(x) = \dfrac{(x+2)(4x+7)}{x^2-7x+6}$.

(a) Find the vertical asymptotes of $r(x)$.

$(x-6)(x-1)=0$; $x=6$, $x=1$

(b) Find the horizontal asymptote of $r(x)$.

$y=4$

(c) Find the zeros of $r(x)$.

$x=-2$, $x=-\frac{7}{4}$

(d) Find the y-intercept of $r(x)$.

$(0, \frac{7}{3})$

14. (8 points) Solve the following rational equation: $\dfrac{1}{5x-7} = \dfrac{8}{3x+4}$.

$$\frac{1(3x+4)-8(5x-7)}{(5x-7)(3x+4)} = 0$$

$$\frac{3x+4-40x+56}{(5x-7)(3x+4)} = 0$$

$$\frac{-37x+60}{(5x-7)(3x+4)} = 0$$

$\triangleright\ -37x+60=0$

$x = \frac{60}{37}$

15. (7 points) Suppose \$8400 is invested in an account paying 2% annual interest and being compounded monthly. How much time is required for the initial investment to triple?

$$3P_0 = P_0\left(1+\frac{.02}{12}\right)^{12t}$$

$$3 = \left(1+\frac{.02}{12}\right)^{12t}$$

$$\ln(3) = \ln\left(1+\frac{.02}{12}\right)^{12t}$$

$$\frac{\ln(3)}{12\ln\left(1+\frac{.02}{12}\right)} = t$$

$\triangleright\ t \approx 54.98$ or roughly 55 yrs.

Came

16. (8 points) Find the inverse of the following matrix:

$$\begin{pmatrix} 3 & 6 \\ 1 & -4 \end{pmatrix}$$

$$\frac{1}{-18}\begin{bmatrix} -4 & -6 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{2}{9} & \frac{1}{3} \\ \frac{1}{18} & -\frac{1}{6} \end{bmatrix}$$

(need to show formula or row reduction technique)

17. (8 points) Using properties of logarithms and assuming all variables positive:

(a) Expand completely: $\log(x^3yz^2)$

$3\log x + \log y + 2\log z$

(b) Solve for x: $2\log(x-7) = 10$.

$\log(x-7) = 5$

$10^5 = x - 7$

$100{,}007 = x$

18. (8 points) Set up and solve a system of equations using 2 variables: Light roast coffee beans cost \$10.00/lb, while dark roast ones cost \$8.00/lb. How much of each type of coffee is needed to create 6 pounds of a mixture that costs \$9.50 per pound?

$$\begin{cases} x + y = 6 \\ 10x + 8y = 6(9.50) \end{cases}$$

$10(6-y) + 8y = 57$

$60 - 2y = 57$

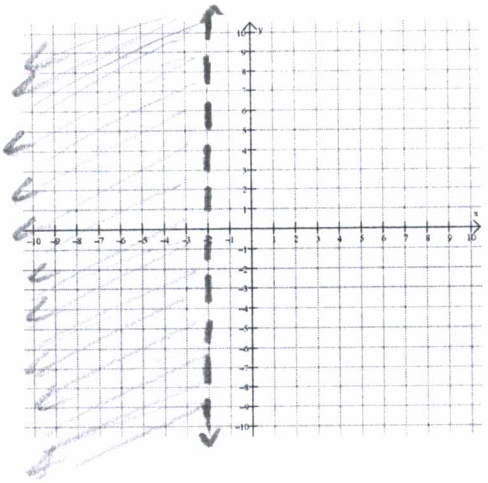
$3 = 2y$,

$y = 1.5$ lbs

$x = 4.5$ lbs

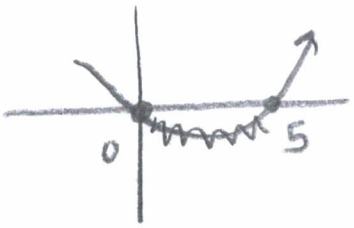
Jiang

19. (6 points) On the grid below, graph the relation $\{(x, y)|x < -2\}$.



20. (6 points) Solve $x^2 - 5x < 0$.

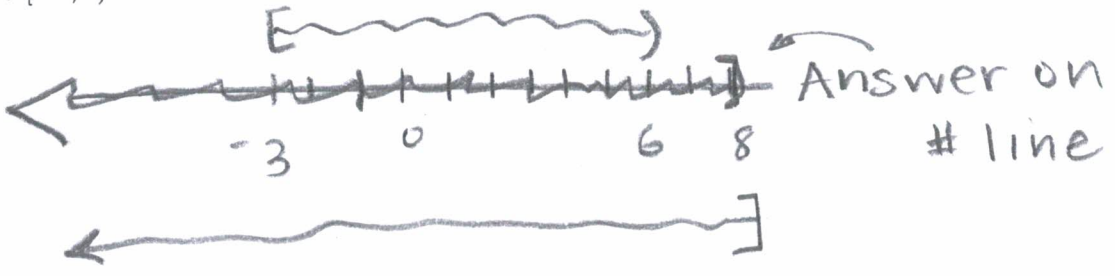
$x(x-5) < 0$



$0 < x < 5$

(# line technique OKAY)

21. (6 points) Find the union. Express answers in **interval notation** and on a **number line**:
$(-\infty, 8] \cup [-3, 6)$



Answer on # line

Interval notation: $(-\infty, 8]$