Name: Key
Recitation Instructor:
Recitation Day and Time:

TRADITIONAL

College Algebra – FINAL EXAM – Spring 2016

Page 1/2 30 pts.	Page 3/4 30 pts.	Page 5/6 30 pts.	Page 7/8 30 pts.	Page 9/10 30 pts.	TOTAL 150 pts

Directions: You will find 20 problems listed below. The point value of each problem is given in parentheses. Please show all your work neatly and box your final answers. No notes or books are allowed. Graphing calculator models above the level of a TI-84 plus are not allowed (in particular, calculators with a built-in CAS and/or QWERTY keyboard are not allowed). You have one hour and fifty minutes to complete this exam.

1. (9 points) Given the points A: $\overset{x_1\ y_1}{(4,1)}$ and B: $\overset{x_2\ y_2}{(-5,1)}$, find the following:

   (a) The slope of the line connecting points A and B.

$$m = \frac{y_2-y_1}{x_2-x_1} = \frac{y_1-y_2}{x_1-x_2}; \quad m = \frac{1-1}{4-(-5)} = \boxed{0}$$

   (b) The distance between points A and B.

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}; \quad d = \sqrt{(-5-4)^2+0} = \sqrt{81} = \boxed{9}$$

(one could also just visually see this is 9 units!)

   (c) The midpoint of A and B.

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right); \quad \left(\frac{4+(-5)}{2}, \frac{1+1}{2}\right); \quad \boxed{\left(-\frac{1}{2}, 1\right)}$$

2. (6 points) Find an equation for the graph x^2, but shifted right 4 units and downwards 6 units.

$$y = (x-4)^2 - 6$$

3. (7 points) Solve for x the equation: $2e^{x+4} - 5 = 13$. Leave exact (don't use a calculator).

$$2e^{x+4} = 18$$

$$e^{x+4} = 9$$

$$\ln(9) = x + 4$$

$$\boxed{x = \ln(9) - 4}$$

4. (8 points) Solve: $|6x + 8| < 12$.

$$-12 < 6x + 8 < 12$$

$-12 < 6x + 8$ and $6x + 8 < 12$

$-20 < 6x$ and $6x < 4$

$-\frac{20}{6} < x$ and $x < \frac{4}{6}$

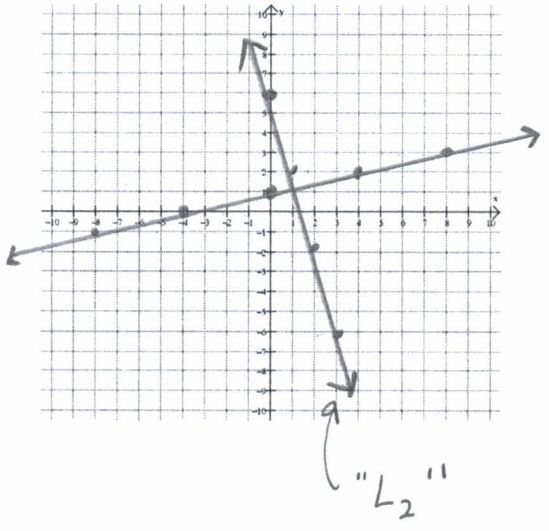
$-\frac{10}{3} < x$ and $x < \frac{2}{3}$

$$\boxed{-\frac{10}{3} < x < \frac{2}{3}}$$

5. (10 points) Graph $-2x + 8y = 8$ and $4x + y = 6$ on the grid below. Label all intercepts clearly as ordered pairs (be sure to find x and y intercepts on both lines). Are the two lines parallel, perpendicular, or neither?

$8y = 2x + 8$

$y = \frac{1}{4}x + 1$ "L_1"



$y = -4x + 6$ "L_2"

Perpendicular

(slopes are negative reciprocals of each other)

6. (5 points) Solve the quadratic equation $3(x-4)^2 - 12 = 0$.

$3(x-4)^2 = 12$

$(x-4)^2 = 4$

$(x-4) = \pm 2$

$x = 4 \pm 2$

$\boxed{x = 6 \text{ or } x = 2}$

7. (7 points) Consider the function $P(x) = -x^2 + 100x + 38,000$. What is the vertex of this function? Show your work with algebra.

(h, k) given by $h = \frac{-b}{2a}$, $k = P(h)$

$h = \frac{-100}{2(-1)} = 50$

$k = P(50) = -(50)^2 + 100(50) + 38000$

$= -2500 + 5000 + 38000$

$= 2500 + 38000$

$= 40,500$

So $(50, 40500)$

8. (8 points) Simplify and write in standard form: $\frac{9-4i}{6+2i}$

$\frac{(9-4i)}{(6+2i)}\frac{(6-2i)}{(6-2i)} = \frac{54 - 18i - 24i + 8i^2}{36 - 4i^2}$

$= \frac{54 - 42i - 8}{36 + 4}$

$= \frac{46 - 42i}{40}$

$= \frac{46}{40} - \frac{42}{40}i$

$= \boxed{\frac{23}{20} - \frac{21}{20}i}$

9. (8 points) Consider $g(x) = 2x^3 + dx$, where d is some external parameter. Answer the following:

(a) Find $g(-2)$. $g(-2) = 2(-2)^3 + d(-2) = \boxed{-16 - 2d}$

(b) Find $g(1)$. $g(1) = 2(1)^3 + d(1) = \boxed{2 + d}$

(c) Find $g(-3)$. $g(-3) = 2(-3)^3 + d(-3) = \boxed{-54 - 3d}$

(d) Find $g(4)$. $g(4) = 2(4)^3 + d(4) = \boxed{128 + 4d}$

10. (~~8~~ 7 points) Consider the polynomial function $p(x) = 2x^3 + 20x^2 + 106x + 348$. Confirm that $x = -6$ is one zero of $p(x)$, and use that fact to find the other zeros (real or complex) of $p(x)$.

-6	2	20	106	348
	↓	-12	-48	-348
	2	8	58	0

$2x^2 + 8x + 58 = 0$

$2(x^2 + 4x + 29) = 0$

Zeros of this are given by:

$$x = \frac{-4 \pm \sqrt{16 - 4(1)(29)}}{2(1)}$$

$$= \frac{-4 \pm \sqrt{-100}}{2} = \frac{-4 \pm 10i}{2} = -2 \pm 5i$$

other zeros are $x = -2 \pm 5i$

11. (7 points) Solve the following rational equation: $\dfrac{x+7}{x+13} = \dfrac{x-3}{5x-1}$.

$$\frac{x+7}{x+13} - \frac{x-3}{5x-1} = 0$$

$$\frac{(x+7)(5x-1) - (x-3)(x+13)}{(x+13)(5x-1)} = 0$$

$$\frac{5x^2 - x + 35x - 7 - (x^2 + 13x - 3x - 39)}{(x+13)(5x-1)} = 0$$

$$\frac{4x^2 + 24x + 32}{(x+13)(5x-1)} = 0 \quad ; \quad \frac{4(x^2+6x+8)}{(x+13)(5x-1)} = 0;$$

$$x^2 + 6x + 8 = 0$$

$$(x+4)(x+2) = 0$$

$$x = -4, \; x = -2$$

12. (8 points) Find $f^{-1}(x)$ when $f(x) = \ln(5 + 2x)$.

Step 1: $y = \ln(5+2x)$

Step 2: solve for x

$$e^y = 5 + 2x$$

$$e^y - 5 = 2x$$

$$\frac{e^y - 5}{2} = x$$

Final Answer:

$$f^{-1}(x) = \frac{e^x - 5}{2}$$

13. (8 points) Using properties of logarithms and assuming all variables positive, answer the following:

(a) Expand completely: $\ln(xy^3z^4)$ $= \ln(x) + \ln(y^3) + \ln(z^4)$

$= \ln(x) + 3\ln(y) + 4\ln(z)$

(b) Condense into a single logarithmic expression the ~~difference~~ sum: $6\log(x) + 5\log(y)$.

$= \log(x^6) + \log(y^5)$

$= \log(x^6 y^5)$

14. (6 points) Suppose \$200 is invested in an account paying 2% annual interest, compounded continuously. Using an exponential growth model, determine the time required for the initial investment to quadruple.

$P(t) = P_0 e^{rt}$

$800 = 200 e^{.02t}$

$4 = e^{.02t}$

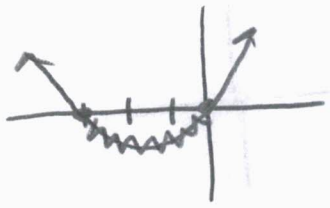
$\ln(4) = .02t$

$t = \frac{\ln(4)}{.02}$; $t \approx 69.3$ yrs

15. (6 points) Solve the quadratic inequality $x^2 + 3x < 0$.

$x^2 + 3x < 0$

$x(x+3) < 0$



Solution: $(-3, 0)$

(# line or case analysis method is fine too)

16. (10 points) Given matrices A and B below, find the product $A^{-1}B$. (Note: Treat a and c as unknown parameters in all your calculations.)

$$A = \begin{pmatrix} 2 & 0 \\ a & -1 \end{pmatrix}$$

$$B = \begin{pmatrix} 4 & 6 \\ c & 8 \end{pmatrix}$$

Final Answer:

$$\begin{bmatrix} 2 & 3 \\ 2a-c & 3a-8 \end{bmatrix}$$

$$A^{-1} = \frac{1}{2(-1)-0}\begin{bmatrix} -1 & 0 \\ -a & 2 \end{bmatrix}$$

$$= \frac{1}{-2}\begin{bmatrix} -1 & 0 \\ -a & 2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1/2 & 0 \\ a/2 & -1 \end{bmatrix}; \quad A^{-1}B = \begin{bmatrix} 1/2 & 0 \\ a/2 & -1 \end{bmatrix}\begin{bmatrix} 4 & 6 \\ c & 8 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2}(4) + 0(c) & \frac{1}{2}(6) + 0(8) \\ \frac{a}{2}(4) + (-1)(c) & \frac{a}{2}(6) + -1(8) \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 \\ 2a-c & 3a-8 \end{bmatrix}$$ Answer

17. (8 points) Consider the rational function $r(x) = \dfrac{(2x+1)(x+19)}{x^2-8x+12}$.

(a) Find the vertical asymptotes of $r(x)$.

$(x-6)(x-2) = 0$; $\boxed{x=6,\ x=2}$

(b) Find the horizontal asymptote of $r(x)$.

$\boxed{y=2}$

(c) Find the zeros of $r(x)$.

$x = -1/2,\quad x = -19$

(d) Find the y-intercept of $r(x)$.

$r(0) = \dfrac{19}{12}$ $\quad \boxed{\left(0, \dfrac{19}{12}\right)}$

18. (8 points) Set up a system and solve the following problem. Light roast coffee beans cost \$4.00/lb, medium roast coffee beans costs \$1.00/lb, and dark roast coffee beans cost \$4.50/lb. If there is twice as much medium roast as there is of the light roast, how much of each type of coffee is needed to create 5 pounds of a mixture that costs \$3.50 per pound? Be sure to indicate the meaning of any variables used in setting up this problem.

Let x = amount of light roast
y = " " medium "
z = " " dark "

$$\begin{cases} x+y+z = 5 \\ 4x+y+4.5z = 5(3.5) \\ y = 2x \end{cases}$$

$$\begin{cases} 3x+z = 5 \\ 6x+4.5z = 17.5 \end{cases} ; \quad \begin{cases} -6x-2z = -10 \\ 6x+4.5z = 17.5 \end{cases}$$

$2.5z = 7.5$; $z = 3$ lbs

$3x+3 = 5$

$3x = 2$

$x = 2/3$ lbs

$y = 4/3$ lbs

19. (7 points) Given $g(x) = x^2 - 5x$ and $h(x) = 4x + 3$, find the following:

(a) $(h + g)(x)$

$= h(x) + g(x) = (4x + 3) + (x^2 - 5x)$

$= \underline{x^2 - x + 3}$

(b) $(hg)(x)$

$= h(x)g(x) = (4x + 3)(x^2 - 5x) = 4x^3 - 20x^2 + 3x^2 - 15x$

$= \underline{4x^3 - 17x^2 - 15x}$

(c) $h(g(x))$

$= h(x^2 - 5x) = 4(x^2 - 5x) + 3$

$= \underline{4x^2 - 20x + 3}$

20. (7 points) Solve the following system using either substitution, elimination, or the matrix inverse method:

$$4x - 3y = 8$$
$$5x + 2y = 4$$

$x = 28/23$

$y = -24/23$

$$\begin{bmatrix} 4 & -3 \\ 5 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -3 \\ 5 & 2 \end{bmatrix}^{-1} = \frac{1}{23} \begin{bmatrix} 2 & 3 \\ -5 & 4 \end{bmatrix} = \begin{bmatrix} 2/23 & 3/23 \\ -5/23 & 4/23 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{2}{23} & \frac{3}{23} \\ \frac{-5}{23} & \frac{4}{23} \end{bmatrix} \begin{bmatrix} 8 \\ 4 \end{bmatrix} = \begin{bmatrix} \frac{16}{23} + \frac{12}{23} \\ \frac{-40}{23} + \frac{16}{23} \end{bmatrix} = \begin{bmatrix} 28/23 \\ -24/23 \end{bmatrix}$$