Name:
Recitation Instructor, Day, Time:

# TRADITIONAL MATH 100 – Exam 2 – October 2017

**Directions:** You will find 16 problems listed below. SHOW ALL WORK! No notes/books/friends are allowed. Graphing calculator models above the level of a TI-84 plus are not allowed. You have one hour to complete this exam.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | TOTAL |
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1. (6 points) Find the solutions and check your answers: $|2x+3| = 16$.

$2x+3=16$ or $2x+3=-16$.

$2x=13$ or $2x=-19$

$x=\frac{13}{2}$ or $x=\frac{-19}{2}$

Both answers work.

Check: $|2(\frac{13}{2})+3| = 16$ ✓

Check: $|2(\frac{-19}{2})+3| = |-16| = 16$ ✓

2. (6 points) Find the solutions to $x^2+8x-5=0$.

$a=1$
$b=8$
$c=-5$

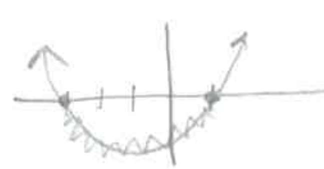
$$x = \frac{-8 \pm \sqrt{64-4(1)(-5)}}{2(1)}$$

$$x = \frac{-8 \pm \sqrt{84}}{2} = \frac{-8 \pm 2\sqrt{21}}{2} = \boxed{-4 \pm \sqrt{21}}$$

3. (6 points) Solve the quadratic inequality $x^2+2x<3$.

$x^2+2x-3<0$

$(x+3)(x-1)<0$.



$\boxed{\text{Solution: } -3 < x < 1}$

* Must have either # line or graphical justification (or a case analysis)

4. (6 points) Given that $x = -4$ is one zero of $p(x) = x^3 + 64$, find all the other zeros, real or complex, of $p(x)$.

$$\begin{array}{r|rrrr} -4 & 1 & 0 & 0 & 64 \\ & \downarrow & -4 & 16 & -64 \\ \hline & 1 & -4 & 16 & 0 \end{array}$$

$x^2 - 4x + 16 = 0$

$$x = \frac{4 \pm \sqrt{16 - 4(1)(16)}}{2(1)} = \frac{4 \pm \sqrt{-48}}{2} = \frac{4 \pm 4\sqrt{3}\,i}{2} = \boxed{2 \pm 2\sqrt{3}\,i}$$

5. (6 points) Suppose a rational function has poles (vertical asymptotes) at $x = 5$ and $x = 3$, zeros at $x = 1$ and $x = -1$, and a horizontal asymptote $y = 2$. Find a possible rational function that has such attributes.

$$r(x) = \frac{2(x+1)(x-1)}{(x-5)(x-3)}$$

6. (6 points) Find the quotient and remainder when $p(x) = x^3 - 5x^2 + 5x + 3$ is divided by $x^2 + 2x + 4$. Write $p(x)$ in the form $d(x)q(x) + r(x)$, where $d(x), q(x)$ and $r(x)$ are the divisor, quotient and remainder, respectively.

$$\begin{array}{r|l} & x - 7 \\ \hline x^2 + 2x + 4 & x^3 - 5x^2 + 5x + 3 \\ & -(x^3 + 2x^2 + 4x) \\ \hline & -7x^2 + x + 3 \\ & -(-7x^2 - 14x - 28) \\ \hline & 15x + 31 \end{array}$$

$$p(x) = \underbrace{(x^2 + 2x + 4)}_{d(x)}\underbrace{(x - 7)}_{q(x)} + \underbrace{(15x + 31)}_{r(x)}$$

7. (6 points) The profit function for selling $x$ units of a certain product is given by $P(x) = -x^2 + 60x + 480$. What number of units generates maximum profit, and, what is the maximum profit? Show your work with algebra. **Remember to answer both parts of the question.**

$(h, k)$: vertex

$h = \frac{-b}{2a} = \frac{-60}{2(-1)} = 30$

$k = P(30) = -900 + 1800 + 480$

$= 900 + 480$

$= \$1380$

30 units generates a max profit of \$1380.

(since $a < 0$, parabola attains a max. @ vertex

8. (6 points) Simplify and write in standard $a + bi$ form: $(-2 + 3i)(-4 + 5i)$

$= 8 - 10i - 12i + 15i^2$

$= \boxed{-7 - 22i}$

Jiang

9. (8 points) Consider the polynomial $p(x) = (2x - 5)(1 - x)(2 - x)(x + 4)$. Circle TRUE or FALSE for each of the statements below.

$(2x-5)(-x+1)(-x+2)(x+4)$

(a) TRUE FALSE $p(x)$ has even degree.

(b) TRUE FALSE $p(x)$ has a negative y-intercept. $P(0) = (-5)(1)(2)(4)$

(c) TRUE FALSE $p(x)$ has positive leading coefficient.

(d) TRUE FALSE As $x \to \infty$, $p(x) \to \infty$.

10. (8 points) A parabola has vertex at $(1, -1)$ and passes through the point $(2, 6)$. What is the equation of the parabola? Write your answer in the form $y = ax^2 + bx + c$.

Jiang

$y = a(x-h)^2 + k$

$6 = a(2-1)^2 - 1$

$6 = a - 1$

$7 = a$

$y = 7(x-1)^2 - 1$

$= 7(x^2 - 2x + 1) - 1$

$= 7x^2 - 14x + 6$

answer

11. (4 points) Using the **REMAINDER THEOREM**, find $p(2)$ when $p(x) = x^4 - 4x^2 + 3x - 2$. Be sure to identify your final answer.

| | | | | | |
|---|---|---|---|---|---|
| 2 | 1 | 0 | -4 | 3 | -2 |
| | ↓ | 2 | 4 | 0 | 6 |
| | 1 | 2 | 0 | 3 | 4 |

$\boxed{p(2) = 4}$

* long division is fine of course

12. (6 points) Simplify $i^{82459}$.

$4 \overline{)82459}$ = 20614 R 3

$i^{82459} = i^{4(20614)+3}$

$= (i^4)^{20614} \cdot i^3$

$= (1) \cdot i^3$

$= \boxed{-i}$

13. (6 points) Solve: $|2x - 5| < 13$.

Jacob

$-13 < 2x - 5 < 13$

$-13 < 2x - 5$ and $2x - 5 < 13$

$-8 < 2x$ and $2x < 18$

$-4 < x$ and $x < 9$

$\boxed{-4 < x < 9}$

14. (6 points) Solve: $|2x + 5| > 9$.

$2x + 5 > 9$ or $2x + 5 < -9$

$2x > 4$ or $2x < -14$

$\boxed{x > 2 \text{ or } x < -7}$

Nicolas

15. (8 points) Consider the parabola $f(x) = -(x+3)^2 + 4$. Answer the following questions. (Drawing a quick sketch of the graph of $f(x)$ may help you.)

(a) What is the domain of $f(x)$?

All reals.

(b) What is the vertex of $f(x)$?

$(-3, 4)$

(c) What is the range of $f(x)$?

$(-\infty, 4]$

(d) What is the axis of symmetry of $f(x)$?

$x = -3$

16. (6 points)Find ALL the zeros of $p(x) = x^4 - x^2 - 72$.

$(x^2-9)(x^2+8) = 0$

$(x-3)(x+3)(x^2+8) = 0$

$\boxed{x = \pm 3, \quad x = \pm 2\sqrt{2}\, i}$