> **Problem**
>
> Compute $\int_0^1 x^2 \, \mathrm{d}x$.

Without/Before FTC

$$\int_0^1 x^2 \, \mathrm{d}x = \lim_{n \to \infty} \sum_{i=1}^n f(x_i)\Delta x \qquad \text{(with } \Delta x = \frac{1}{n} \text{ and } x_i = 0 + i\Delta x = \frac{i}{n}\text{)}$$

$$= \lim_{n \to \infty} \sum_{i=1}^n \frac{i^2}{n^2} \cdot \frac{1}{n}$$

$$= \lim_{n \to \infty} \frac{1}{n^3} \sum_{i=1}^n i^2 \qquad \text{(sum of squares formula)}$$

$$= \lim_{n \to \infty} \frac{1}{n^3} \cdot \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{1}{6} \lim_{n \to \infty} 1 \cdot (1 + \tfrac{1}{n})(2 + \tfrac{1}{n})$$

$$= \boxed{\frac{1}{3}}$$

With/After FTC

$$\int_0^1 x^2 \, \mathrm{d}x = \frac{1}{3} \, x^3 \Big|_0^1 = \frac{1}{3}(1 - 0) = \boxed{\frac{1}{3}}$$